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TIME EFFECTS IN SPECTROPHOTOMETRY.

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1. According to Schwarzschild, if a photographic emulsion be exposed to light of intensity I for a time t, the optical density D of the image (or any other measure of photographic effect) will be given by an equation of the form

$$D = \text{a function of } It^p,$$

where p is a constant for any particular plate and has come to be known as the Schwarzschild index.

Schwarzschild's hypothesis would, if universally true, have a very simplifying effect on spectrophotometric technique. For it can easily be shown, as a consequence of the hypothesis, that if a characteristic curve be constructed from a series of calibration exposures (*i.e.* from a series of exposures for which the exposure time is constant and the ratios of the intensities are known) it can be applied, without error, to the measurement of an unknown light ratio which is photographed on the same plate using a different exposure time. Further, with the additional hypothesis that the index p does not vary with wave-length, it can be shown that in spectrophotometry of the continuous spectrum the spectra of two sources of light can be photographed with two different exposure times, the plate calibrated with a third exposure time, and the difference of gradient between the two light sources deduced, without error, by treating the measures just as if the exposure times were identical.

Unfortunately the Schwarzschild hypothesis has been shown by the work of Kron and others to be merely a first approximation, valid over a very limited range. It is now realised that there is an optimal value of the intensity for which the energy It necessary to produce a given photographic

effect is a minimum. An analytical expression for the law of photographic action was suggested by Kron and modified by Halm.

The modified formula is

$$\text{density} = \text{a function of } It/\psi,$$

where

$$\psi = \tfrac{1}{2}[(I/I_0)^{\alpha} + (I/I_0)^{-\alpha}].$$

I_0 being the optimal intensity and α a constant.

This formula has been found to agree fairly well with experiment, and it must now be accepted that the range of validity of the Schwarzschild approximation is so limited as to introduce errors in spectrophotometric measurements which involve the use of different exposure times. The object of the present paper is to investigate the nature and magnitude of these errors.

2. Introduce variables x and y defined as follows :—

$$x = \log I,$$
$$y = \log (It),$$

the logarithms being taken to base 10.

The Schwarzschild hypothesis may then be written in the form

$$f(D) = (1-p)x + py$$

so that the curves of equal density are a family of straight lines on the (xy) diagram. On the other hand the Kron-Halm law becomes

$$f(D) = y - \log \cosh \{\alpha M(x - x_0)\},$$

where

$$x_0 = \log I_0 \quad \text{and} \quad M = \log_e 10.$$

The equal density curves are now logarithmic catenaries. For each of these curves y is a minimum for $x = x_0$.

Curves of equal density on the (xy) diagram have been determined for a variety of emulsions by members of the Eastman Kodak Research Staff,* who have called such curves "reciprocity failure curves." It appears from their experimental results that the Kron-Halm expression is a fair approximation. The times corresponding to optimal intensity for moderate densities (say $D = 1{\cdot}0$ to $D = 1{\cdot}5$) † are of the order of one second for ordinary temperatures.

In astronomical spectrophotometry we are nearly always concerned with weak light fluxes, and so we are working on the low-intensity side of the optimum. Also, the ranges of the variables in any particular investigation

* See papers on "The Relation between Time and Intensity in Photographic Exposures," by L. A. Jones and his colleagues in *J. Opt. Soc. Amer.*, **7**, 1107 ; **11**, 319 ; **12**, 321 ; **13**, 443 ; and **14**, 223. These papers are reprinted in an abbreviated form in *Abridged Scientific Publications from the Research Laboratories of the Eastman Kodak Company*, vols. 8 to 11.

† By density is meant optical density, *i.e.* the logarithm of the ratio of incident to transmitted light when a beam is passed through the developed image.

are limited in practice, and would be covered by time and light ratios of 100 to 1. Further, in the case of spectrophotometry of the continuous spectrum the range of density permitted is usually very restricted, so that in this kind of work we are concerned with a strip of the (xy) plane which surrounds an equal density curve. The areas of the (xy) plane with which the astronomer is concerned are thus comparatively limited, and a mere casual inspection of the equal density curves obtained by the Kodak investigators shows that, over the limited areas in question, the family of curves can be represented approximately by the system of parabolas

$$y + F(x) = c,$$

where

$$F(x) = ax - bx^2,$$

a and b being constants.

On detailed examination it is found that this approximation is a very good one. On testing a reciprocity failure curve given by J. H. Webb * for a high-speed panchromatic emulsion exposed to the mercury line at λ 4360 it was found that over a time range of 13^s to 8500^s the equal density curve for $D = 1{\cdot}0$ was represented by the parabolic form to an accuracy of less than 0·01 in log (It). This emulsion was selected for test on account of the enhanced curvature of its reciprocity failure curve. The curves given by Webb for three other emulsions were also tested, and a similar accuracy of fit was found.

The present paper is concerned with applications to practical spectrophotometric technique, and accordingly it is preferable to employ a parabolic representation, such as the above, which is capable of high accuracy over the limited ranges of the variables employed in practice, rather than resort to more complicated analytical expressions such as the Kron-Halm formula, which is intended to cover a much greater area of the (xy) diagram, but which may not be capable of yielding high accuracy. It will be shown that the parabolic form leads to simple formulæ, which can be employed in practice to correct for time effect.

We accordingly adopt as the law of photographic action

$$f(D) = y + ax - bx^2.$$

Put

$$\log t = \tau.$$

Then $y = x + \tau$ and the law of photographic action becomes

$$f(D) = (1 + a)x - bx^2 + \tau. \qquad (1)$$

We proceed to the derivation of formulæ exhibiting spectrophotometric time effects.

3. *Line Profile Spectrophotometry.*—Suppose that a line profile is photographed with exposure time t, and that the plate is calibrated with a different exposure time, which, for convenience, we shall adopt as the unit of time.

* *J. Opt. Soc. Amer.*, 23, 162.

Put $\log t = \tau$ as before. For a given density D, the logarithmic light-intensity x is erroneously deduced as x' where

$$(1+a)x' - bx'^2 = f(D),$$

whereas the true value x is given by

$$(1+a)x - bx^2 + \tau = f(D).$$

By subtraction $(1+a)(x'-x) - b(x'^2 - x^2) - \tau = 0$.

Solving this equation for x' and expanding in ascending powers of b, we get, after some algebra,

$$x' = x + \frac{\tau}{1+a} + \frac{b}{(1+a)^2}\left[2x\tau + \frac{\tau^2}{1+a}\right] + \text{higher powers of } b.$$

The quantity b will, for most emulsions, be so small that its square can be neglected (see the numerical results in the next section).* Also we are concerned in practice with the measurement of light ratios, *i.e.* with the measurement of the *differences* of logarithmic intensities. We accordingly get

$$\Delta x' = \Delta x\left[1 + \frac{2b}{(1+a)^2}\tau\right].$$

Multiplying both sides by 5/2 so as to get the familiar magnitude scale there results

$$\Delta m' = \Delta m[1 + \epsilon\tau], \qquad (2)$$

where Δm is the true magnitude difference, $\Delta m'$ is the magnitude difference derived from the calibration curves, and

$$\epsilon = \frac{2b}{(1+a)^2}.$$

4. *Some Numerical Applications.*—(*a*) In connection with the Greenwich Colour Temperature work, a determination of the quantity ϵ was carried out some time ago. A number of plates were exposed with the telescope, using the method of full and half apertures, thereby photographing a light-ratio of 2 : 1, *i.e.* a magnitude difference of $0^m.75$.† These plates were calibrated with a multiple slit spectrograph, using a uniform exposure time of 20^s for the calibration exposures. The telescope exposure times varied from 20^s to 540^s. The results were analysed in the form

$$\Delta m' = \Delta m[\alpha + \epsilon\tau],$$

* The terms in b^2 involve x^2, and in order to prevent them from coming appreciable the unit of I, *i.e.* the zero from which x is measured, should be chosen within the range of values over which the parabolic fit is being made. It may be noticed that a change of the zero point of x by an amount r will alter the coefficient a by an amount $2br$. This does not affect the result (2) in which second order terms have been neglected. In fitting a parabola to a reciprocity failure curve it is convenient to take as the origin of x the middle point of the range of values of x considered, and this has been done in the case of Webb's curves, from which numerical values of ϵ have been deduced in the next section.

† For fuller details see *The Observatory*, 59, 11, 1936 January.

where $\Delta m'$, as above, denotes the magnitude derived from the calibration characteristic curves, Δm ($= 0^m{\cdot}75$) is the true magnitude difference, and τ is the logarithm of the ratio of the exposure time at the telescope to the calibration exposure time ($= 20^s$). The constant α was inserted so as to take into account the difference of conditions between the telescope and calibration exposures.*

The value found for ϵ was

$$\epsilon = 0{\cdot}075 \pm 0{\cdot}011.$$

This result is a combined result derived from measures at different wave-lengths. Analysis of the data showed that there was no variation, within the limits of experimental error, of the quantities α and ϵ with wave-length.

The plates used for the work were Ilford "Hydrogen Alpha" developed in dilute pyro-soda for 10 minutes at 60° F. The wave-lengths used ranged from λ 4140 to λ 6820. The broadening of the spectra photographed with the slitless spectrograph attached to the telescope was effected by an electric motor and cam, the period of the complete double stroke being about 5 seconds.

(*b*) In connection with the Greenwich absolute colour temperature work,† a series of exposures, each of 20 minutes duration, were made with the multiple slit calibration spectrograph on a Kodak Standard Acetylene Burner and on a standard filament lamp. The values, in magnitudes, of one of the slit differences, were measured on forty of these plates, using the calibration curves obtained from two-minute exposures with the same multiple slit spectrograph. The mean value of the slit difference obtained in this way was $0^m{\cdot}850 \pm 0^m{\cdot}004$, whereas the true value was $0^m{\cdot}772$. Hence putting in (2) $\Delta m = 0{\cdot}772$, $\Delta m' = 0{\cdot}850$ and $\tau = 1$, there results

$$\epsilon = 0{\cdot}101 \pm 0{\cdot}005.$$

This result is a mean value derived from measures at various wave-lengths. The results for the separate wave-lengths are as follows :—

λ	ϵ
4470	0·093 ± ·005
4785	0·114 ± ·006
5307	0·095 ± ·008
5780	0·107 ± ·009
6423	0·097 ± ·008

* In the telescope exposures the broadening was secured by trailing the image and was thus intermittent, whereas the calibration exposures were obtained by continuous exposures on a tungsten filament lamp. The constant α may be regarded as defining what are equivalent exposure times as between the slitless spectrograph on the telescope and the multiple slit calibration spectrograph. It is rather curious that the value found for α should be quite close to unity (the actual result was $\alpha = 0{\cdot}99$). This means that a characteristic curve derived from exposures with the telescope, using a *total* exposure time of 20^s, will be practically the same as that derived from the 20^s continuous exposures with the calibration spectrograph. In view of the fact that, for the telescope exposures, individual grains of the emulsion are only exposed to light for a small fraction (less than a tenth) of the total exposure time, this was a wholly unexpected result.

† *M.N.*, 94, 6, 496, 1934.

It is concluded from these values that there is no appreciable variation of ϵ with wave-length.

The plates used in this work were Imperial Panchromatic "B" developed in dilute pyro-soda at 60° F.

(*c*) Values of ϵ have been derived for four emulsions whose reciprocity curves have been given by Webb in the paper cited in section 2 above. Each of these curves refers to light of wave-length 4360 A. The plates were developed in Pyrogallol developer at 68° F. The results are given in the following table; the ranges given in the second column are the limits of exposure time over which the parabolic formula was fitted to the curves.

Values of ϵ deduced from Webb's Curves

Emulsion	Range of Exposure Time	ϵ
	s s	
Low Speed Blue Sensitive	12 to 1260	0·012
High Speed Blue Sensitive I	5 to 650	0·045
High Speed Blue Sensitive II	6·5 to 620	0·073
High Speed Panchromatic	13 to 8500	0·184

(*d*) R. O. Redman,* using Ilford Ordinary Process plates, has found no variation, greater than 1 per cent., in characteristic curve over the range of exposure time 0·5 to 15 minutes. For these plates ϵ is accordingly very small. They are evidently similar in behaviour to the low-speed blue sensitive plates used by Webb, the reciprocity curve being practically a straight line (so that the Schwarzschild approximation is a very good one) over the range in question.

It would be tempting to conclude that this is a characteristic of all process plates, but Jones and Webb † have pointed out that there is no general correlation between the shape of the reciprocity failure curves and other characteristic attributes of the emulsion such as speed, grain, sensitisation, etc. They also remark that the effect of development upon reciprocity failure is slight. These remarks should serve as a warning to workers in spectrophotometry that the behaviour of each emulsion used must be tested. It will be noticed that if anyone used the high-speed panchromatic emulsion quoted by Webb, for which $\epsilon = 0{\cdot}18$, and neglected time effects entirely, serious errors would result. The use of emulsions for which ϵ is of this order of magnitude should be avoided.‡

5. *Spectrophotometry of the Continuous Spectrum.*—Suppose that, for a comparison between two stars, x_1 and x_2 are the logarithmic light fluxes and τ_1 and τ_2 the logarithmic exposure times. Suppose, further, that the plate is calibrated with a third exposure time, which will be adopted as the unit of time.

* *The Observatory*, 58, 126.

† *J. Soc. Mot. Pict. Eng.*, 23, 142, 1934; also *Kodak Abridged Scientific Publications*, No. 531.

‡ It should be noticed that for such emulsions the neglected second order terms in ϵ^2 are no longer negligible if an accuracy of 1 per cent. is contemplated and time ratios of the order of 10 to 1 are being used.

The gradient difference between the two stars is deduced from the measures just as if all the exposure times were identical. It is required to find the error.

Let x_1' and x_2' be the erroneous logarithmic fluxes deduced from the calibration curve. Then, as in section 3,

$$x_1' = x_1(1+\epsilon\tau_1) + \frac{\tau_1}{(1+a)}[1+\tfrac{1}{2}\epsilon\tau_1],$$

$$x_2' = x_2(1+\epsilon\tau_2) + \frac{\tau_2}{(1+a)}[1+\tfrac{1}{2}\epsilon\tau_2].$$

By subtraction we get

$$(x_1'-x_2') = (x_1-x_2)[1+\tfrac{1}{2}\epsilon(\tau_1+\tau_2)] + \tfrac{1}{2}\epsilon(\tau_1-\tau_2)(x_1+x_2) + \frac{\tau_1-\tau_2}{1+a} + \frac{1}{2}\frac{\epsilon}{1+a}(\tau_1^2-\tau_2^2).$$

We shall assume that ϵ does not vary with wave-length, an assumption which appears to be justified within the limits of experimental error (see previous section). We shall also assume for the present that a is independent of λ.

Multiplying both sides of the above equation by $M = \log_e 10$ and differentiating with respect to $1/\lambda$ we get

$$-\Delta\phi' = -\Delta\phi[1+\tfrac{1}{2}\epsilon(\tau_1+\tau_2)] + \epsilon(\tau_1-\tau_2)\frac{d}{d(1/\lambda)} \cdot \tfrac{1}{2}M(x_1+x_2),$$

where $\Delta\phi'$ is the gradient difference (in the sense first star *minus* second) between the two stars as deduced from the measures, ignoring time effects, and $\Delta\phi$ is the true gradient difference.*

Now

$$\frac{d}{d(1/\lambda)} \cdot \tfrac{1}{2}M(x_1+x_2) = -(\phi_m-\phi_0),$$

where ϕ_m is the mean value of ϕ for the two stars, and ϕ_0 is the value of ϕ for which the blue and red light fluxes incident on the plate are equal.†

We thus get

$$\Delta\phi' = \Delta\phi[1+\tfrac{1}{2}\epsilon(\tau_1+\tau_2)] + \epsilon(\tau_1-\tau_2)(\phi_m-\phi_0),$$

or, since we are neglecting squares of ϵ,

$$\Delta\phi = \Delta\phi'[1-\tfrac{1}{2}\epsilon(\tau_1+\tau_2)] - \epsilon(\tau_1-\tau_2)(\phi_m-\phi_0). \qquad (3)$$

In the above we have assumed that a is independent of λ. Now when $\epsilon = 0$, so that the law of photographic action reduces to the Schwarzschild

* The notation used is that given in a Greenwich publication, "Observations of Colour Temperatures of Stars," 1932. The quantity ϕ is related to the colour temperature T by the relation $\phi = \frac{c_2}{T}/(1-e^{-c_2/\lambda_m T})$ where $c_2 = 14320$ microns-degrees, and λ_m (in microns) is the mean wave-length of the spectral range used.

† This is a departure from the notation previously used. In the Greenwich publication the symbol ϕ_0 was employed to denote an entirely different quantity.

hypothesis, we have $a=(1-p)/p$ where p is Schwarzschild's index. Experimental investigations,* based on the Schwarzschild hypothesis, have shown that p is independent of wave-length or nearly so, and the same experimental data, when interpreted in the light of the present theory, show that $da/d\lambda$ is either negligible or at any rate of the same order of magnitude as ϵ. We then find that in place of (3) we have

$$\Delta\phi=\Delta\phi'[1-\tfrac{1}{2}\epsilon(\tau_1+\tau_2)]-\epsilon(\tau_1-\tau_2)(\phi_m-\phi_0'), \qquad (3')$$

where

$$\phi_0'=\phi_0-\frac{M}{\epsilon(1+a)^2}\cdot\frac{da}{d(1/\lambda)},$$

M being, as before, the modulus of common logarithms.

(3′) is identical in form with (3), the only difference being the substitution of ϕ_0' for ϕ_0.

To use formula (3′) for correcting for time effects it is necessary to know the value of ϕ_0' as well as that of ϵ. ϕ_0' can be determined by observing the apparent difference of gradient of a star against itself, different exposure times being used, and the light flux for the longer exposure being reduced by a coarse diffraction grating or some similar device. Observations of this kind were made some years ago with the 30-inch reflector at Greenwich, and have been published in the Greenwich Colour Temperature publication.† Working on the simple Schwarzschild approximation the results were used to infer that there was no variation of the Schwarzschild index p with wave-length, and consequently that there were no errors arising from time effect. This result is now seen to be erroneous, and the negligible value of the gradient difference obtained is now seen to be the result of the fact that the observations were made on stars of type A, for which, owing to the characteristics of the optical train employed, the blue and red fluxes on the plate were approximately equal, and so $\phi_m-\phi_0'$ was small. The observations can now be used to infer the value of ϕ_0', which turns out to correspond to a gradient (measured relatively to the zero of the Greenwich system) of $+0{\cdot}16$. The resulting corrections to the measured gradients are usually very small.‡ This is in part due to the circumstance that for the majority of the stars on the programme $\phi-\phi_0'$ is small.

6. *Concluding Remarks.*—The formulæ (2) and (3′) above are of a simple character, and provide a ready means of correcting spectrophotometric observations for time effects, provided the requisite constants are known. They are put forward in the hope that they will be of some assistance to those whose work is of such a nature as to make them hesitate to impose the restriction of using identical exposure times in all comparisons. Such a restriction, although ideal, sometimes results in such a diminution of output as to be impracticable. When unequal times are used it appears from the

* See, for example, p. 17 of the Greenwich Colour Temperature publication.

† *Loc. cit.*

‡ As a result of the aluminising of the subsidiary mirrors of the Yapp 36-inch reflector a change in ϕ_0 has recently taken place with consequential increased corrections for the majority of the stars on the current programme.

above that it is necessary to investigate the character of the emulsion used and to determine the value of the constant ϵ. The work of Redman, quoted above, is of particular interest, as it shows that for a certain blue sensitive emulsion time effects can be ignored over a large range. For work in the blue region of the spectrum, provided high speed is not required, an emulsion such as this should, of course, be employed in preference to one for which the value of ϵ is appreciable. It would be a great boon to workers in spectrophotometry if a high-speed panchromatic emulsion possessing a similarly negligible curvature of the reciprocity failure curve could be discovered.

In this paper temperature variations have been ignored. The recent work of Webb * has shown that the shape of the reciprocity failure curves is effected by temperature, and that for very low temperatures of the order of $-75°$ C. the curves are practically linear (so that $\epsilon = 0$ for this temperature). It does not appear from Webb's results that for ordinary working temperatures serious error will be introduced by neglecting the variation of ϵ with temperature. The recent work, however, does emphasise the desirability of calibrating plates at approximately the same temperature as that at which they were exposed.

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THE INTERNAL CONSTITUTION OF THE PLANETS.

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(Communicated by Professor E. A. Milne)

Introduction.—The present paper is concerned with the following problem : Is it possible from purely theoretical considerations (1) to deduce a relation connecting the radius and mass of a planet and (2) to predict the maximum radius for a *cold* † (stellar or planetary) body ? The maximum value for the radius, as estimated from observational data, is about one-tenth of the solar radius, a value about equal to the diameter of Jupiter.‡ The problem that we take up in this paper is the same as considered by Majumdar and the author in a recent paper § (referred to hereafter as KM I), but the method here employed—which is the same as that given in Article 1 of a

* *J. Opt. Soc. Amer.*, 25, 4, 1935.

† We are using here the word *cold* in the technical sense. Matter will be referred to as *cold* if its temperature and density are such that any *free* electrons present constitute a *degenerate* gas in the sense of Fermi-Dirac statistics.

‡ Attention to this very significant fact has been drawn by Professor Russell (as reported in *Observatory*, 58, 260, 1935). The present paper owes its origin to the above remarks of Professor Russell.

§ Kothari and Majumdar, *Nat. Acad. Sci. U.P. India*, 6, 57, 1936 ; also *Nature*, 137, 157, 1936.

previous paper * on "White Dwarfs and Electro-static Correction"—is, I believe, simpler and more straightforward, and the final result (expressible by a simple formula) is in better accord with observation.

Our starting-point in KM I as well as the present paper is the usual theory of the white dwarf stars, but with one significant modification in that the degree of ionisation in degenerate matter is not treated as fixed, *i.e.* μ the mean molecular weight *per* free electron is not treated as independent of the mass of the star, but is regarded, as it must be, as a variable, and its dependence on the mass M of the configuration is taken account of. Thus we introduce no new hypothesis, but merely free the usual theory from a serious limitation and then work out its consequences. The theory in this form not only explains—as the usual theory does—the structure of the white dwarf stars, but also gives an *insight into the structure of the planets and predicts a maximum radius for a cold body.*

In KM I the dependence of μ on the mass of the configuration is evaluated by applying the theory of pressure-ionisation. In the case of non-degenerate matter the degree of ionisation is determined essentially by temperature and to some extent by pressure in accordance with Saha's formula. For degenerate matter the temperature concept is relegated to the background and the degree of ionisation is determined essentially by the pressure (or density), and for this reason it is spoken of as pressure-ionisation. The theory of pressure-ionisation has been discussed in previous papers † and also in KM I.

In this paper we make use of the *virial* theorem, and by including the electrostatic potential energy with the gravitational and total kinetic energy of the configuration automatically take account of the dependence of the degree of ionisation in degenerate matter on the mass of the configuration. In section 1 the relevant results of the usual white dwarf theory are summarised. In section 2 we derive the fundamental formula of this paper and discuss its applications in section 3.

1. In the case of the white dwarf stars the researches beginning with Fowler and later developed extensively by Milne and others have shown that the essential features of their internal constitution can be explained by an application of Fermi-Dirac statistics. A white dwarf star is characterised by a low luminosity, a high effective temperature and a very large mean density. In a preliminary theoretical treatment it is usual to assume the luminosity of a white dwarf to be zero, *i.e.* to regard it as a *black dwarf*—a term originally due to Fowler. This procedure is not as unreasonable as it may appear at first sight, for, as has been shown elsewhere,‡ the small luminosity of a white dwarf has little effect on the calculated values of its radius and mean density.

If we have a star of mass M which has ceased to radiate and is composed of ionised matter degenerate in the sense of Fermi-Dirac statistics, then, as has been shown by several authors, the radius R of the star is connected with

* Kothari, *Phil. Mag.*, **12**, 665, 1931.

† Kothari and Majumdar, *Astr. Nachr.*, **244**, 65, 1931 ; Swirles, *Proc. Roy. Soc.*, **141**, 561, 1933.

‡ Kothari, *M.N.*, **93**, 70, 1932.

its mass by the relation * (neglecting relativity effects, this being justified so long as the mass is not much larger than the Sun)

$$R = \frac{5(\omega^0{}_{3/2})^{1/3}K}{2^{7/3}\pi^{2/3}G\mu^{5/3}}\frac{1}{M^{1/3}}$$

$$= 2{\cdot}79 \times 10^9 \frac{1}{\mu^{5/3}}\left(\frac{\odot}{M}\right)^{1/3} \text{cm.}, \qquad (1)$$

where $\odot$ is the mass of the Sun, μ is the mean molecular weight *per* free electron and $K/\mu^{5/3}$ denotes the "degenerate gas constant." † If ρ denotes the density of the material, then $\rho/\mu m_H$—m_H being the mass of the hydrogen atom—gives the number of *free electrons* per unit volume. It may be noted here that the usual definition of μ is average weight per particle, but as in degenerate gas the "heavy particle" is to be consistently ignored, we define μ as the molecular weight *per free electron*. We shall speak of it as the "electron molecular weight" to distinguish it from the usual definition.‡

If ρ_m denotes the mean density of the star, then from (1) we have

$$\rho_m = \frac{96\pi}{125(\omega^\circ{}_{3/2})}\left(\frac{G}{K}\right)^3 \mu^5 M^2 \qquad (2)$$

$$= 2{\cdot}14 \times 10^4 \mu^5 \left(\frac{M}{\odot}\right)^2 \text{gm./cm.}^3.$$

Further, if ρ_c denotes the central density, then ρ_c and ρ_m are related by the relation (given in Milne's notation)

$$\frac{\rho_m}{\rho_c} = 3(\omega^\circ{}_{3/2})/(\sigma_{3/2})^{3/2} = 1/5{\cdot}99. \qquad (3)$$

For any *fully ionised* element $\mu = A/Z$, where A is its atomic weight and Z its atomic number, and therefore μ is nearly 2 (*e.g.* for iron $A = 56$, $Z = 26$, $\mu = 2{\cdot}15$) except for ionised hydrogen, for which $\mu = 1$. The value A/Z represents the minimum value for μ; in the case of partial ionisation of the element μ will be greater. In the case of hydrogen of abundance such that

* Milne, *M.N.*, **92**, 610, 1932. The notation used in (1) above is slightly different from that of Milne's paper in that our μ denotes the molecular weight on the chemical scale whereas Milne's μ is in grams, and therefore his μ equals our μm_H, m_H being the mass of the hydrogen atom. The "degenerate gas constant" $K/\mu^{5/3}$ used here is Milne's K. This is done to show explicitly the dependence on μ (see Kothari, *M.N.*, **93**, 70, 1932).

† The "degenerate gas constant" is defined by the relation :

$$\text{pressure of degenerate gas} = \frac{8\pi h^2}{15m}\left(\frac{3n}{8\pi}\right)^{5/3} = \frac{8\pi h^2}{15m}\left(\frac{3}{8\pi m_H}\right)^{5/3}\frac{\rho^{5/3}}{\mu^{5/3}} = \frac{K\rho^{5/3}}{\mu^{5/3}},$$

n in above is the number of *free* electrons per unit volume.

‡ This distinction between the two definitions for μ is often not stated explicitly and this may lead to some confusion. Thus Chandrasekhar in *M.N.*, **95**, 208, 1935, equation (3), implies by μ the molecular weight per free electron, but refers to it simply as molecular weight. Similarly μ in Fairclough's paper (*M.N.*, **95**, 585, 1935) implies the electron molecular weight.

there are x atoms of hydrogen to one atom of the other element (following a usual practice we shall take the other element to be iron in numerical work) the value of μ for complete ionisation of the material will be given by

$$\mu = \frac{A+x}{Z+x} \sim 1 + \frac{Z}{Z+x}. \tag{4}$$

In the case of Sirius B, which has a mass of $0{\cdot}85\ \odot$, the observed radius is about 22×10^8 cm. If we assume the stellar matter to be (on an average) composed of iron—and we shall speak of it as assumption F—then assuming complete ionisation ($\mu = 2{\cdot}15$) we get from (1) $R = 8{\cdot}2 \times 10^8$ cm. This value is smaller than the observed one by a factor of about 3. No doubt in formula (1) we assumed the luminosity to be zero, but, as already remarked, even if we took account of the luminosity, the calculated value of R will not be appreciably modified.* As will be seen later, in the case of the planets also the calculated radius on assumption F comes out smaller than the observed radius, and the discrepancy again is of a factor of about 3. However, the observed value of R can be made to agree with the calculated value by introducing the hypothesis of hydrogen abundance. Thus, to take the case of Sirius B, the observed R agrees with the calculated value if we take $\mu = 1{\cdot}2$, *i.e.* assume the stellar material to contain a large proportion of hydrogen (about 70 per cent. by weight).†

Equation (1), therefore, is in agreement with the observed mass-radius relation for white dwarfs, provided we make the two assumptions that (i) the stellar matter contains a good proportion of hydrogen, and (ii) the material is completely ionised. Assumption (i) is in harmony with recent researches on the constitution of stars in general; one cannot, of course, justify it on purely theoretical grounds. Assumption (ii) is, however, on a different footing. We should expect that in a complete theory of white dwarfs this result, instead of being assumed, should follow naturally from the theory. The theory developed in this paper is an attempt in this direction.

We have so far discussed the case of *cold* bodies of mass nearly that of the Sun.‡ We now ask the question as to what will happen for masses $M << \odot$.

* For the case of the model white dwarf $M = \frac{1}{2}\odot$, $L = 10^{31}$ erg/sec., $L/M = 10^{-2}$ erg/sec. grm. considered in *M.N.*, **93**, 70, 1932, R is found to be $1{\cdot}03 \left(\frac{2{\cdot}1}{\mu}\right)^{5/3} 10^9$ cm. If L be assumed zero, then R will have the value $1{\cdot}02 \left(\frac{2{\cdot}1}{\mu}\right)^{5/3} 10^9$ cm. The effect of luminosity in this case is to increase R by about 3 per cent. For Sirius B, $L = 10^{31}$ erg/sec., and the effect of luminosity on the calculated radius would in all probability be not much more than that for the model white dwarf.

† The hypothesis of hydrogen abundance has been applied to the white dwarf stars by several investigators, *e.g.* Mitra, *Zeits. für Astrophys.*, **4**, 329, 1932, and Strömgren, *Zeits. für Astrophys.*, **7**, 222, 1933. For o_2 Eridani B the proportion of hydrogen required is much less than for Sirius B.

‡ The white dwarf stars for which data exist so far have all masses near about that of the Sun—in no case is it less than about $\odot/5$.

The relation (1) shows that if we assume μ to be constant (independent of M), R continues to increase as M decreases. But this inference stands in contradiction to observation, for in the case of planets which are *cold* bodies of mass much less than the Sun (their cores correspond, one may say, to ideal black dwarfs) a larger mass is associated, not with a smaller, but with a larger radius. Further, for any planet the observed radius is larger than the radius calculated from (1) for $\mu = 2{\cdot}15$ by a factor of the order of 100.* Thus, if we restrict ourselves to the assumption of μ being independent of M, then for $M << \odot$, equation (1) leads to absurd results. However, there is no *a priori* justification for this assumption, and as it leads to wrong results it must be discarded. In the next section we develop the theory so as to be free from the above restriction and work out its consequences.

2. Let us consider a collection of particles moving under their own mutual forces. If the force between any two particles varies inversely as the square of the distance between them, then it is a well-known result† (easily established classically and also wave-mechanically) that

$$2T + W = 0, \tag{5}$$

where T denotes the total kinetic energy of all the particles in the assembly and W is the total potential energy.

We shall apply this result to a *cold* spherical aggregate of mass M composed of matter of atomic weight A and atomic number Z. We proceed to estimate T and W for such a mass.

In estimating the total kinetic energy T we include all electrons,‡ bound as well as free. As is now well known from the investigations of Fermi, Thomas and others, the *bound* electrons in an atom can be treated as forming a degenerate gas, and as the matter is *cold* the free electrons (if they be present) will also be degenerate.§ We can assign to each atomic nucleus with its Z electrons (in general some of these electrons will be *bound* and the others *free*) an element of volume or spherical cell of radius a which can be expressed in terms of the density ρ by the relation

* It is possible that one may feel it not quite reasonable to apply (1) to planets and to criticise it on the account that the predicted radius is wrong. It should be remarked that we nowhere assert whether (1) ought or ought not to apply to planets —we merely examine whether it does or does not. We find as a matter of fact that it does not apply when μ is taken independent of M, and this merely serves as an impetus for working out a theory which will be free from the above limitation regarding μ. The theory so developed, surprisingly enough, predicts all the essential features of planetary structure.

† This is called the *virial* theorem. For a proof from wave mechanics, see Frenkel, *Wave Mechanics*, vol. ii, p. 30. The general result for force varying as the nth power of the distance is $2T = (n+1)W$. The sign of the force (attraction or repulsion) is immaterial and it may vary from particle to particle. The state of zero potential energy is taken to be that when the particles constituting the assembly are all dispersed to infinity.

‡ For *cold* matter the total kinetic energy is almost entirely due to electrons, the nuclei making negligible contribution.

§ If the matter was not *cold* the free electrons would be non-degenerate and their kinetic energy determined by the temperature.

$$\frac{a^3}{\gamma}\frac{\rho}{Am_H}=1, \tag{6}$$

where m_H is the mass of the hydrogen atom and γ is a factor of the order unity. It will lead to no serious error in estimating the kinetic energy if we assume the Z electrons as uniformly distributed in the cell.* The usual formula for the kinetic energy of a degenerate electron gas is

$$E_0=\frac{3}{10}\frac{h^2}{m}\left(\frac{3n}{8\pi}\right)^{2/3}N, \tag{7}$$

where n is the electron concentration, *i.e.* number of electrons per unit volume, and N is the total number of electrons. Substituting Z for N and $\frac{Z\gamma}{a^3}$ for n, we obtain for the kinetic energy per cell T' the expression

$$T'=\frac{3}{10}\frac{h^2}{m}\left(\frac{3}{8\pi}\frac{Z\gamma}{a^3}\right)^{2/3}Z. \tag{8}$$

Substituting for $\frac{\gamma}{a^3}$ from (6) we have

$$T'=\frac{3}{10}\frac{h^2}{m}\left(\frac{3}{8\pi}\frac{Z\rho}{Am_H}\right)^{2/3}Z. \tag{9}$$

In estimating to a first approximation the value of T we may neglect the variation of density from the boundary towards the centre of the configuration and take for ρ in (9) the mean density ρ_m of the configuration, †

$$\rho_m=M\Big/\frac{4\pi}{3}R^3, \tag{10}$$

where R is the radius of the configuration. Multiplying T' by the number of cells in mass M which is M/Am_H, we obtain for the total kinetic energy T

$$\begin{aligned}T&=\frac{3}{10}\frac{h^2}{m}\left(\frac{3}{8\pi}\frac{Z\rho_m}{Am_H}\right)^{2/3}\frac{M}{Am_H}Z\\&=\beta\left(\frac{Z}{A}\right)^{5/3}\frac{1}{R^2}M^{5/3},\end{aligned} \tag{11}$$

where

$$\begin{aligned}\beta&=\frac{3}{5}\frac{1}{2^{1/3}}\left(\frac{3}{8\pi}\right)^{4/3}\frac{h^2}{m}\frac{1}{m_H{}^{5/3}}\\&=5{\cdot}71\cdot10^{12}.\end{aligned} \tag{12}$$

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* See, for example, Feinberg, *Physikalische Zeit. der sowjetunion*, 8, 409, 1935.

† The density distribution will be very nearly polytropic with index 3/2. For this polytrope the mean density is 1/6th the central density, and except close to the boundary there is no wild departure between the actual and the mean density. If necessary the variation in density could be taken account of by a simple numerical integration (see Kothari, *Phil. Mag.*, *loc. cit.*, § 5). However, it will not affect essentially the results presented here.

The potential energy W of the configuration is the sum of the electrostatic energy W_S and the gravitational energy W_G. It is no simple matter to calculate W_S accurately, but an approximate value for it can easily be obtained. The total electrostatic energy of the configuration will be obtained by multiplying the electrostatic energy of a single cell by the number of cells. Assuming that the Z electrons are uniformly distributed in the cell of radius a, the cell having at its centre a nucleus of charge $+Ze$, the potential $U(r)$ at any point inside the cell and distant r from its centre will be, after the cell has been built up to radius r,

$$U(r) = (Ze - \tfrac{4}{3}\pi r^3 ne)\frac{1}{r}, \tag{13}$$

where n is the electron concentration.

Therefore the potential energy W_S' of the cell will be given by

$$\begin{aligned} -W_S' &= \int_0^a \frac{(Ze - \frac{4}{3}\pi r^3 ne)}{r} \cdot 4\pi r^2 ne\, dr \\ &= \frac{9}{10}\frac{Z^2e^2}{a} \end{aligned} \tag{14}$$

and hence

$$-W_S = -W_S'\frac{M}{Am_H} = \frac{9}{10}\frac{Z^2e^2}{a}\frac{M}{Am_H}. \tag{15}$$

Substituting for a from (6) and taking γ equal to unity * we have

$$\begin{aligned} -W_S &= \tfrac{9}{10}Z^2e^2 \cdot \frac{\rho_m^{1/3}}{(Am_H)^{4/3}}M \\ &= \alpha_1\frac{M^{4/3}}{R}\frac{Z^2}{A^{4/3}}, \end{aligned} \tag{16}$$

where

$$\alpha_1 = \tfrac{9}{10}\left(\frac{3}{4\pi}\right)^{1/3}\frac{e^2}{m_H^{4/3}} = 6{\cdot}47\ 10^{12}. \tag{17}$$

In calculating both T and W_S we have neglected the effect of the variation of density inside the configuration and have treated it as if the density were uniform and equal to the mean density ρ_m. The gravitational energy of a sphere of mass M and uniform density is easily shown to be

$$-W_G = \tfrac{3}{5}G\frac{M^2}{R} = \alpha_2\frac{M^2}{R}, \tag{18}$$

where

$$\alpha_2 = \tfrac{3}{5}G = 4{\cdot}00\ 10^{-8}. \tag{19}$$

Substituting the values for T, W_S and W_G in (5), which expresses the virial theorem, we have

$$2\beta\left(\frac{Z}{A}\right)^{5/3}\frac{M^{5/3}}{R^2} = \alpha_1\frac{Z^2}{A^{4/3}}\frac{M^{4/3}}{R} + \alpha_2\frac{M^2}{R}. \tag{20}$$

* The exact value of γ depends on a number of factors—particularly the lattice arrangement of the atoms—but in any case it will be of the order unity.

From this we obtain for R

$$R = \frac{2\beta\left(\frac{Z}{A}\right)^{5/3} M^{5/3}}{\alpha_1 \frac{Z^2}{A^{4/3}} M^{4/3} + \alpha_2 M^2} \tag{21}$$

$$= \frac{\frac{2\beta}{\alpha_2}\left(\frac{Z}{A}\right)^{5/3} \frac{1}{\odot^{1/3}}}{1 + \frac{\alpha_1}{\alpha_2} \frac{Z^2}{A^{4/3}} \frac{1}{\odot^{2/3}} \left(\frac{\odot}{M}\right)^{2/3}} \left(\frac{\odot}{M}\right)^{1/3}, \tag{22}$$

or

$$R = \frac{\frac{2\beta}{\alpha_1} \frac{1}{(ZA)^{1/3}} \odot^{1/3}}{1 + \frac{\alpha_2}{\alpha_1} \frac{A^{4/3}}{Z^2} \odot^{2/3} \left(\frac{M}{\odot}\right)^{2/3}} \left(\frac{M}{\odot}\right)^{1/3}. \tag{23}$$

We have obtained a relation connecting the radius and mass of the configuration. Equation (23) is the fundamental equation of the present paper. *It shows that as* M *increases from zero upwards,* R *also increases from zero upwards, attains a maximum value and then begins to decrease, and finally vanishes as* $M \to \infty$.

Differentiating R with respect to M we find that—
(i) the radius R has its maximum value for $M = M_0$, where M_0 is given by

$$M_0 = \left(\frac{\alpha_1}{\alpha_2}\right)^{3/2} \frac{Z^3}{A^2} \tag{24}$$

or, substituting the numerical values of α_1 and α_2,

$$M_0 \sim 1{\cdot}04\ 10^{-3} \frac{Z^3}{A^2} \odot, \tag{25}$$

and (ii) this maximum value of the radius is

$$R_{\max} = \frac{\beta}{\sqrt{\alpha_1 \alpha_2}} \frac{Z^{2/3}}{A} \sim 1{\cdot}12\ 10^{10} \frac{Z^{2/3}}{A} \text{cm}. \tag{26}$$

Expressing equation (21) in terms of $R_{\max}$ and M_0 we have

$$\frac{R}{R_{\max}} = \frac{2(M/M_0)^{1/3}}{1 + (M/M_0)^{2/3}}. \tag{26'}$$

When $M >> M_0$, equation (23) approximates to

$$R \sim \frac{2\beta}{\alpha_2} \left(\frac{Z}{A}\right)^{5/3} \frac{1}{M^{1/3}} = 2{\cdot}28\ 10^9 \left(\frac{Z}{A}\right)^{5/3} \left(\frac{\odot}{M}\right)^{1/3} \text{cm}. \tag{27}$$

and for $M << M_0$ it approximates to

$$R \sim \frac{2\beta}{\alpha_1} \frac{1}{(ZA)^{1/3}} M^{1/3} = 2{\cdot}22\ 10^{11} \left(\frac{1}{ZA}\right)^{1/3} \left(\frac{M}{\odot}\right)^{1/3} \text{cm}. \tag{28}$$

We notice at once that equation (27) is just the same as equation (1) of the usual white dwarf theory, except that μ in (1) is now replaced by Z/A and the numerical factor in (1) is a little larger than in (27). The small difference in the numerical factors is due to the fact that (1) is deduced for a polytropic distribution of density, whereas in deriving (27) the density has been assumed uniform. The fact that our theory predicts for μ in (1) the value Z/A shows that for configurations of mass $M >> M_0$—and the known white dwarfs satisfy this condition—the stellar material is completely ionised. *In the usual white dwarf theory this is taken as an assumption; here it follows naturally from the theory.* The second case of $M << M_0$ corresponds to planetary masses. A larger mass in this case is associated with a larger radius.

For $M > M_0$ gravitational energy predominates over the electrostatic energy, whereas for $M < M_0$ the opposite is the case.*

3. Before we can compute numerical results and compare them with observation we must make some assumption about the chemical composition of the material, *i.e.* assume some reasonable values for A and Z. The numerical results given here are worked out on two alternative assumptions :

(i) *Assumption F.*—The material is assumed to be on an average iron with $Z = 26$ and $A = 55{\cdot}8$.

(ii) *Assumption H.*—The material is assumed to be hydrogen with $Z = 1$ and $A = 1$.

It is found that for a given mass M the calculated radius on assumption F is smaller than the observed value, whereas on assumption H the calculated radius is larger.

Fig. 1 exhibits the results of our calculations. The full-line curves which bound the shaded region on the upper and lower sides represent the theoretical mass-radius relation given by equation (23). The upper curve corresponds to the case when the material is assumed to be all hydrogen, the lower curve is for iron. The observed (M, R) values for the planets and the white dwarf stars are indicated in the figure. It is to be particularly noted that all the observed (M, R) values, except for the white dwarf Procyon B, fall within the region between the H and Fe curves. This is indeed a satisfactory result. Roughly speaking, assumption H gives for the lighter planets a radius about 4 times the observed value, and on assumption F the calculated radius is about 3 times too small. For the heavier planets, Jupiter and Saturn, assumption H gives a value about $1\frac{1}{2}$ times too large and assumption F a value 3 times too small compared to the observed value. It appears therefore possible that by taking a suitable proportion of hydrogen and iron, *i.e.* by a suitable choice regarding the chemical composition of the material, the theoretical value could be made to fit with the observed value,

* In my 1931 paper on the white dwarfs and the electrostatic correction (*loc. cit.*) I observed that the electrostatic correction becomes predominant for any white dwarf, if such exists, with $M < \odot/10$. However, I had entirely overlooked the applicability of these results to the case of planets, it never having occurred to me that a planet was an (ideal) "black dwarf."

and we could derive information about the average chemical composition of the planetary material.* However, we shall not attempt here the calculations for a mixture of elements, as in this case the evaluation of the electrostatic energy proves to be a difficult matter. All the same the theory does bring out the essential points that for white dwarfs a smaller mass means a

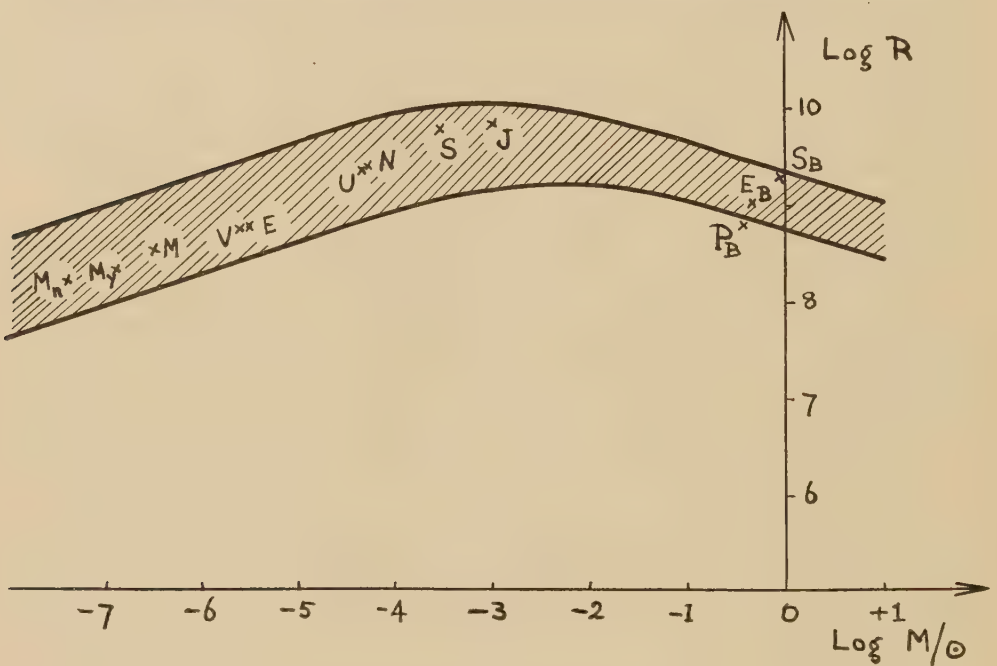

FIG. 1.—*The full-line curves bounding the shaded region are theoretical. The upper curve is for Hydrogen and the lower for Iron.*

M_n = *Moon.*	E = *Earth.*	$\mathcal{J}$ = *Jupiter.*
M_y = *Mercury.*	U = *Uranus.*	S_B = *Sirius B.*
M = *Mars.*	N = *Neptune.*	E_B = o_2 *Eridani B.*
V = *Venus.*	S = *Saturn.*	P_B = *Procyon B.*

larger radius, whereas for the planetary bodies a smaller mass corresponds to a smaller radius ; *and that there exists a maximum radius for a "cold" body.* The values of R (maximum) and M_0 are at once found from (26) and (25), and are given in the following table. The values under the heading "Observed" have been roughly estimated from the run of the observed (M, R) values.

The observed values lie between the two theoretical values. The value

* As the planets have in all probability been formed out of the Sun, a reasonable assumption about their chemical composition would be to take it to be the same as assumed for the Sun in theories dealing with its internal constitution. The non-hydrogen elements would be represented by the Russell mixture and the proportion of hydrogen would be about one-third by weight (Strömgren, *Zeits. für Astrophysik*, 4, 118, 1932 ; 7, 322, 1933). Preliminary calculations seem to indicate that the theoretical (M, R) value in this case would be in fairly good accord with observation.

	R_{max} in cm.	$M_0/\odot$
Calculated on assumption F	$1{\cdot}77 \times 10^9$	$5{\cdot}84 \times 10^{-3}$
Calculated on assumption H	$1{\cdot}12 \times 10^{10}$	$1{\cdot}04 \times 10^{-3}$
Observed	8×10^9	$1{\cdot}6 \times 10^{-3}$

of R_{max} calculated on assumption F is smaller than the observed value by a factor of about $3\frac{1}{2}$ and the value calculated on assumption H is about $1\frac{1}{2}$ times larger.

In conclusion it may be observed that a satisfactory theory of the structure of stellar masses composed of degenerate matter must introduce no *ad hoc* assumption about the degree of ionisation in degenerate matter, but should be capable of predicting the state of ionisation and how it varies with the mass of the configuration. Such a theory in its preliminary aspects is easily developed, and while it leaves the usual results for the white dwarf stars essentially unaltered, it fortunately gives an insight into the general features of planetary structure and *predicts a maximum radius for a "cold" body.*

It is a pleasure to record my grateful thanks to Professor E. A. Milne for his kind interest in this work. My thanks are also due to Dr. R. C. Majumdar for friendly discussion.

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FRAUNHOFER INTENSITIES IN THE INFRA-RED REGION 6600–8900 A.

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The main object of extending the table of Fraunhofer intensities already published * into the infra-red is to obtain more information on multiplet intensities, *f*-values, absorption coefficient, and atomic abundance in the Sun. These subjects, however, will be discussed elsewhere, and only the intensity observations are given here.

Photometry.—The spectra were obtained in the first order of the 21-foot Rowland concave grating of the Solar Physics Observatory, Cambridge. The plates used were Ilford Infra-red, and Eastman *U*, *N* and *P* types ; they were not ammoniated. There were at least three plates to cover the entire range from 6600–8900 A. Three methods of standardisation were employed : (*a*) For one set of plates use was made of a series of solar spectra

* C. W. Allen, *Mem. C.S.O. Canberra*, No. 5, 1934.

whose intensities were varied by introducing neutral perforated screens which had themselves been standardised with a thermo-couple. Generally the exposure times of the standardisation spectra were shorter than those of the spectra to be photometered, and with this method it has been necessary to assume that the plate characteristic curves were parallel for time ratios up to 45 : 1. (*b*) A few plates were standardised by means of the grating astigmatism. The intensity distribution of light across a concave grating spectrum may be computed from the length of the slit and the length of the grating lines.* This makes a convenient though not very reliable method of standardisation. (*c*) Finally a complete set of plates was obtained standardised with a tube sensitometer. An infra-red filter was used in the sensitometer, and the smallness of the sensitivity range for each make of plate ensures that the standardisation wave-length is nearly the same as that of the examined spectra.

Speaking generally, photometry in the infra-red is more difficult than in the visible part of the spectrum. The plate grain is coarser, and the longer exposure times introduce photometric difficulties. Individual observations are not as consistent as in the visible spectrum, but the tabulated results are the mean of several observations and should be reliable.

Equivalent Breadths.—Contours have been plotted and areas measured to obtain the equivalent breadths of about 100 lines in various parts of the infra-red spectrum. These values were plotted against central intensity, and the curves so obtained used to convert the central intensities of the rest of the lines into equivalent breadths. An attempt was made to correct for blending (chiefly with atmospheric lines), but this was difficult, and blended lines are not very reliable. For many spectra a wide slit was employed so that any difference in the sharpness of the Fraunhofer lines should not greatly affect those equivalent breadths that have been determined from central intensities.

The equivalent breadths are in the mean about 10 per cent. smaller than Mulders's values.†

Results.—The results are given in Table I, in which the first four columns are taken from the *Revised Rowland Table of Solar Wave-lengths*, 1928. The fifth column contains new estimates of intensity recently made in connection with the extensive study of the infra-red solar spectrum at Mount Wilson. I am indebted to Mr. H. D. Babcock and Miss C. E. Moore for the use of this new material prior to its publication. The wave-lengths in this forthcoming publication will be revised to accord with the new standards,‡ and will differ slightly from those given here, particularly in the region 6600–7330 A.

The sixth column in Table I contains the equivalent breadths derived from the measured central intensity r_m. The next column gives the equivalent breadths found from the areas of the line contours, while the last

* E. C. C. Baly, *Spectroscopy*, 3rd ed., vol. i, p. 165.

† G. F. W. Mulders, "Aequivalente breedten van Fraunhoferlijnen in het zonnespectrum," *Dissertation*, Utrecht, 1934.

‡ H. D. Babcock, C. E. Moore and W. P. Hoge, *Ap. J.*, **83**, 103, 1936.

TABLE I

Wave-length	El.	Low E.P. Volts	Intensity		Equivalent Breadth in Milli-angstroms		
			Old R.R.	New B and M	from r_m	from Contour	Other Observers
							Mu
6604·609	*Sc*+	1·351	1	3	33	39	39
6606·988	*Ti*+	2·052	−1*N*	−1	...		13
6608·053	*Fe*	2·269	0	1	16		18
6609·126	*Fe*	2·548	3	4	70	60	75
6625·048	*Fe*	1·007	0	1	13		16
6627·569	*Fe*	4·529	0	3	28		28
6633·436			0	0	22		
6633·767	*Fe*		2	4	68		
6634·132	*Fe*	4·775	0	1	30		
6635·146	*Ni*	4·400	0*N*	1	26		29
6639·726	*Fe*	4·587	0	1	16		
6643·648	*Ni*	1·669	5	7	102	96	98
6646·975	*Fe*	2·597	−1	0	...		9
6653·920	*Fe?*	4·126	−1	0	...		11
6663·255	*Fe*	4·539	1	1	...		
6663·470	*Fe*	2·414	3	5	85		
6667·749	*Fe*		−1	−2	...		11
6668·409			−2*N*	−3	...		6
6669·319	*Cr?*	4·157	−2*N*	−2	...		6
6678·007	*Fe*	2·681	5	8	130	121	143
6680·164			−2	−1	...		6
6696·041	−*Al*	3·129	1*N*	2	32		41
6698·678	*Al*	3·129	0	1	20		24
6699·145	*Fe*		−1	0	8		
6703·585	*Fe*	2·747	1	3	36	37	39
6704·509	*Fe*	4·199	−2	−2	...		6
6705·120	*Fe*		1	4	44	45	49
6710·332	*Fe*	1·478	0	1	12		12
6713·053	*Fe*	4·587	1	2	26		33
6713·754	*Fe*	4·775	1	1	22	23	24
6715·395	*Fe*	4·588	1	2	28	27	32
6716·261	*Fe*		0	1	15		18
6717·697	*Ca*	2·697	5	8	118	116	120
6719·640			−2*N*	−1*NN*	...		14
6721·853			2	3	41	42	48
6724·694			−2*N*	−2	...		8
6725·373	*Fe*	4·086	0	1	19		24
6726·682	*Fe*	4·587	2	3	52	49	55
6729·028	*Fe*		−1	−1	...		9
6733·162	*Fe*	4·618	1	2	26		

Wave-length	El.	Low E.P. Volts	Intensity		Equivalent Breadth in Milli-angstroms		
			Old R.R.	New B and M	from r_m	from Contour	Other Observers
6737·987			0	1	21		
6741·638			0*N*	1	13		
6743·136	*Ti*	0·896	1	1	13		
6750·173	*Fe*	2·414	3	5	82	74	
6752·725	*Fe*	4·618	1	3	37	35	
6755·614	*Fe*		0	1	12		
6767·793	*Ni*	1·818	4	6	85	80	
6771·071	*Co*	1·874	0*d*	2	18		
6772·330	*Ni*	3·642	2	4	53	58	
6777·415	*Fe*		−1	−1	11		
6786·869	*Fe*	4·173	1	2	25		
6793·283	*Fe*		−1	0	11		
6793·638	*Fe*	4·775	0*N*	1	13		
6804·020	*Fe*	4·632	0	1	22		
6804·307	*Fe*		0	0	16		
6806·866	*Fe*	2·716	1	3	35		
6810·276	*Fe*	4·587	3	4	53	53	
6814·971	*Co*	1·947	0	1	14		
6820·385	*Fe*		2	4	40	38	
6828·610	*Fe*	4·618	2	4	58	53	
6837·025	*Fe*		0	1	17		
6838·845	*Fe*		0	1	25		
6839·847	*Fe*	2·548	1	1	27	27	
6841·355	*Fe*	4·587	3	4	72	69	
6842·055	*Ni*	3·642	0	1	27	30	
6842·701	*Fe?*		1	2	40	44	
6843·672	*Fe*		3	4	64	62	
6848·578			0*N*	0	15		
6855·180	*Fe*	4·539	3	5	79	79	
6855·735	*Fe*	4·587	0	0	17		
6857·268	*Fe*		0	1	21		
6858·165	*Fe*	4·588	2	4	54	54	
6861·957	*Fe*	2·414	0	0	13		
6862·511	*Fe*	4·539	1	1	27		
6880·637	*Fe*	4·136	−1	−1	12		
6881·726	*CrFe*	3·423	0	0	27		
6882·530	*Cr*	3·423	1	0	29		
6883·072	*Cr*	3·423	1	0	29		
6898·307	*Fe*	4·202	−1	−2	11		
6914·578	*Ni*	1·942	4	3	83		

Wave-length	El.	Low E.P. Volts	Intensity		Equivalent Breadth in Milli-angstroms		
			Old R.R.	New B and M	from r_m	from Contour	Other Observers
6916·694	*Fe*	4·136	2	2	51		
6925·288	*Cr*	3·434	1	0	40		
6926·108	– *Ti?*	3·567	0	–1	17		
6933·052	*Fe*	4·173	0*N*	0	15		
6945·224	*Fe*	2·414	4	3	74		
6951·260	*Fe*	4·539	1	2	60		
6960·337	*Fe*		–1	–2	11		
6971·953	*Fe*	3·005	–1	1	15		
6975·458	*Fe*		0	1	17		
6976·533	*Si*		1	2	39		
6978·420	*Cr?*	3·449	1*N*	3	58		
6978·875	*Fe*	2·474	2	4	76		
6979·812	*Cr*	3·449	1	2	33		
6988·544	*Fe*	2·394	0	1	26		
6999·905	*Fe*	4·086	1	4	50		
7003·590	*Si*		1	6	57		
7005·909	*Si*		1	5	...		
7007·987	*Fe*	4·160	0	1	21		
7016·081	*Fe*		1	6	(84)		
7022·972	*Fe*	4·173	2	4	62		
7024·078	*Fe*		0	1	27		
7024·654	*Fe*	4·539	1	2	...		
7024·886	*Ni?* –	4·518	0	1	51		
7030·037	*Ni*	3·527	0	1	18		
7032·334			1	3	22		
7034·920			2*N*	7	63		
7038·238	*Fe*	4·199	1	5	54		
7038·787	*Fe*	4·238	0	2	32		
7068·425	*Fe*	4·892	2	7	48		
7071·867	*Fe*	4·587	0*N*	3*N*	32		
7084·989	*Co*	1·874	2*N*	6	49		
7086·739	*Fe*	5·064	–1	2	18		
7090·404	*Fe*	4·212	2	7	64	60	
7095·417	*Fe*	4·191	0	3	20		
7110·921	*Ni*	1·927	1	3	30		
7122·221	*Ni*	3·527	4	10	103		
7130·939	*Fe*	4·199	3	9	97	82	
7133·000	*Fe*		1	4	42	38	
7142·530	*Fe*	4·934	0	3	39		
7145·321	*Fe*	4·587	1	4	41		

Wave-length	El.	Low E.P. Volts	Intensity		Equivalent Breadth in Milli-angstroms		
			Old R.R.	New B and M	from r_m	from Contour	Other Observers
7148·163	*Ca*	2·697	3*N*	12	137	121	
7155·649	*Fe*	4·988	0*N*	3	42		
7165·568	*Si*	5·846	1*N*	5*d*?	80		
7189·158	*Fe*		−1	2	40		
7202·211	*Ca*	2·697	1*N*	5	111		
7207·408	*Fe*	4·136	1	5	98		
7212·448	*Fe*	4·934	−1	1	24		
7219·699	*Fe*		0	3	47		
7221·218	*Fe*	4·539	0	3	33		
7222·400	*Fe*$^+$	3·872	−2*N*	2	18		
7244·868	*TiFe*		−1	3	60		
7250·661	*Si*	5·595	−1*N*	5	60		
7251·732	*Ti*	1·424	−2*N*	1	22		
7284·851	*Fe*		−1	3	40		
7288·756	*Fe*	4·202	−1	4	63		
7289·199	−*Si*	5·595	0*N*	5	85		
7306·583	*Fe*	4·160	−1	3	34		
7307·954	*Fe*		−1*d*?	3	57		
7320·689	*FeFe*$^+$		1	4	79		
7326·176	*Ca*	2·920	0	9	(100)	101	
7355·893	*Cr*	2·877	1	5	54		
7389·391	*Fe*	4·283	2	9	105	115	
7393·610	*Ni*	3·590	2	8	95	88	
7400·184	*Cr*	2·887	1	7	69		
7405·790	*Si*	5·589	1	9	86		
7409·355	*Ni*	3·780	1	8	76		
7411·158	*Fe*	4·265	1	8	107	100	
7414·512	*Ni*	1·977	1	7	66	62	
7415·957	*Si*	5·591	1	7	82		
7418·674	*Fe*		0	5	43		
7422·286	*Ni*	3·620	1	8	88	81	
7423·509	*Si*	5·594	1	9	95	96	
7440·915	*Fe*	4·892	1	5	53		
7445·755	*Fe*		2	9	119	125	
7462·342	*Cr*	2·901	2	9	97		
7491·652	*Fe*	4·283	1	5	56	62	
7495·076	*Fe*	4·202	2	12	134	156	
7507·276	*Fe*	4·396	1	4	57	48	
7511·030	*Fe*	4·160	2	12	166	171	
7522·782	*Ni*	3·642	1	7	74		

Wave-length	El.	Low E.P. Volts	Intensity		Equivalent Breadth in Milli-angstroms		
			Old R.R.	New B and M	from r_m	from Contour	Other Observers
7525·115	*Ni*	3·620	1	7	69	74	
7531·152	*Fe*	4·353	2	8	91		
7555·608	*Ni*	3·831	2	9	89	93	
7568·906	*Fe*	4·265	1	8	70	73	
7574·046	*Ni*	3·816	1	7	64		
7583·796	*Fe*	3·005	1	8	80	86	
7586·025	*Fe*	4·294	2	9	118	134	
7616·977	*Ni*	3·639	2	5	100		
7619·212	*Ni*	3·663	1	5	50		
7657·605	*Mg*	5·086	2	8	81		
7661·196	*Fe*	4·238	1	6	68		
7664·297	*Fe*		1	9	84		
7664·880	*AtmK*	0·000	6	(25)	...		Mu
7680·267	*Mn*	5·467	1	7	74	86	82
7691·562	*Mg*	5·728	2	8	76	103	120
7698·977	*K*	0·000	5	10	154	135	152
7710·367	*Fe*	4·202	1	4	60	59	75
7711·737	Fe^+	3·887	1	4	42	45	47
7714·309	*Ni*	1·927	3	7	100	102	115
7715·577	*Ni*	3·683	1	4	46		49
7723·211	*Fe*	2·269	1	4	37	35	43
7727·616	*Ni*	3·663	3	7	86	78	97
7742·722	*Fe*	4·970	2	9	86	110	103
7748·284	*Fe*	2·936	2	8	100		105
7748·894	*Ni*	3·690	2	7	84		84
7751·117	*Fe*	4·970	0	4	44		45
7771·954	*O*	9·106	2	7	62	60	78
7774·177	*O*	9·106	2	6	56	51	74
7775·394	*O*	9·106	1	6	48	45	65
7780·567	*Fe*	4·454	3	9	112	113	135
7788·933	*Ni*	1·942	2	8	84	72	95
7797·587	*Ni*	3·881	2	6	72	71	80
7799·995			0	4	48		51
7807·915	*Fe*	4·970	1	5	54	52	60
7832·207	*Fe*	4·416	2	9	126		154
7836·139			0	3	54		44
7849·984			1	5	50		62
7855·392	*Fe*	5·042	0	3	27		36
7912·869	*Fe*	0·855	1	2	45		
7918·383	*Si*	5·928	1	5	82		

Wave-length	El.	Low E.P. Volts	Intensity		Equivalent Breadth in Milli-angstroms		
			Old R.R.	New B and M	from r_m	from Contour	Other Observers
7932·351	*Si*	5·938	2	5	100		
7933·12	*Cu*	3·770		1	25		
7937·149	*Fe*	4·294	3	9	(181)	160	
7941·096	*Fe*		1	2	47		
7944·001	*Si*	5·958	2	7	112	125	
7945·857	*Fe*		2	9	(159)	167	
7959·152	*Fe*	5·011	−1	0	21		
7994·488	*Fe*		1	2	44		
7998·953	*Fe*	4·353	2	8	144		
8028·321	*Fe*	4·454	1	5	(88)		
8046·054	*Fe*	4·396	2	8	(154)		
8047·623	*Fe*	0·855	1	3	61		
8075·153	*Fe*	0·911	0	2	28		
8080·58			0	−2	26		
8085·172	*Fe*	4·426	2	9	(174)		
8092·637	*Cu*	3·800	1	2	38		
8093·938	*Co?*	4·004	−1	−2	20		
8096·872	*Fe*	3·005	0	1	24		Ri
8183·258	*Na*	2·093	3	4?	...		195
8194·836	*Na*	2·905	2	8	...	253	263
8207·747	*Fe*	4·426	1	3	77	79	
8213·040	*Mg*	5·728	3	4	116		
8220·381	*Fe*	4·302	2	5	(198)	194	
8232·321	*Fe*	4·396	0	3	85		
8239·136	*Fe*	2·414	0	2	47	37	
8248·136	*Fe*	4·353	0	3	70		
8248·800			0	3	72		
8293·519	*Fe*		−1	2	57		
8327·060	*Fe*	2·188	2	6	(210)		
8331·926	*Fe*	4·368	1	3	142		
8339·408	*Fe*	4·416	0	4	80		
8346·130	*Mg*	5·920	0*N*	5	110		
8360·794	*Fe*	4·454	0	3	52		
8365·640	*Fe*		1	4	65		
8377·862	*Ti*	0·822	−1	1	31		
8387·781	*Fe*	2·167	3	9	(215)	186	
8401·396	*Fe*		−1	1	25		
8412·358	*Ti*	0·815	0	3	37		
8422·923	*Fe*		−1	1	20		
8424·131	*Fe*		−1	2	29		

Wave-length	El.	Low E.P. Volts	Intensity		Equivalent Breadth in Milli-angstroms		
			Old R.R.	New B and M	from r_m	from Contour	Other Observers
8426·514	*Ti*	0·822	0	3	43		
8434·966	*Ti*	0·845	1	4	57		
8435·653	*Ti*	0·832	1	3	54		
8439·577	*Fe*	4·529	0	5	72		
8446·350	*O*	9·479	0	5*N*	66		
8446·728	*O*	9·479	−1	3	26		Mu
8468·417	*Fe*(*Ti*)		2	7	164	134	123
8471·738	*Fe*		−1	3	44		
8496·997	*Fe*	4·588	0	4	39		38 Da
8498·060	Ca^+	1·685	12	(20)	...	1170	1180 940
8501·556	*Si*	5·846	−1	1	...		28
8502·228	*Si*	5·846	0	3	60		48
8514·082	*Fe*	2·188	1	6	127	116	107
8515·121	*Fe*	3·005	0	5	78	73	73
8526·675	*Fe*	4·892	0	3	68	69	63
8536·161	*Si*	6·154	0	3	...		37
8538·023			−1	2	...		20
8539·90			−2 ?	0	...		24
8542·132	Ca^+	1·693	16	(25)	...	3120	3440 3190
8546·228			−2	2 ?	...		26
8556·790	*Si*	5·846	1	7	98	112	135
8582·271	*Fe*	2·977	1	5	71	78	72
8592·971	*Fe*	4·934	−1	3*N*	40	52	60
8595·967			−1	3	34	37	43
8598·835	*Fe*	4·368	−1	4	48	58	63
8610·609	*Fe*	4·416	−1	2	....		42
8611·813	*Fe*	2·833	1	6	110	100	105
8613·944			−1	2	...		42
8616·280	*Fe*	4·892	0	3	37	50	49
8621·617	*Fe*	2·936	1	5	76	90	71
8648·472	*Si*		2	9	142	167	159
8662·170	Ca^+	1·685	15	(30)	...	2430	2640 2640
8674·757	*Fe*	2·819	1	6	131	139	126
8686·369			−1	3	37		
8688·641	*Fe*	2·167	2	10	...	260	248
8699·462	*Fe*	4·934	1	4	65		
8710·396	*Fe*	4·892	0	4	108		
8712·673			−1	3	43		
8713·212	*Fe*	2·936	−1	3	56		
8717·833			0	5	72		

Wave-length	El.	Low E.P. Volts	Intensity		Equivalent Breadth in Milli-angstroms		
			Old R.R.	New B and M	from r_m	from Contour	Other Observers
8728·023	– *Si*		0	4	88		
8736·040	*Mg*	5·920	1	12	150	157	
8742·466	*Si*	5·846	1	6	112	103	
8752·025	*Si*	5·846	1	7	121	122	
8757·199	*Fe*	2·833	1	9	102	82	
8763·976	*Fe*	4·632	1	4	106		
8772·880	*Al*	4·004	0	3	70		
8773·909	*Al*	4·005	0	4	99		
8784·45	*Fe*	4·934	– 1	1	30		
8790·454	*FeSi*		1	5	117	136	
8793·347	*Fe*	4·588	1	5	138	139	
8804·639	*Fe*	2·269	0	3	51		MG
8806·776	*Mg*	4·327	4	12	...	450	512
8824·235	*Fe*	2·188	2	9	...	184	
8838·449	*Fe*	2·846	1	6	102	110	
8846·743	*Fe*	4·988	– 1	3	41		
8862·564	*Ni*	4·072	0	3	54		
8866·943	*Fe*	4·529	1	8	(165)	199	
8868·445	*Fe* –	3·005	1	3	...		
8912·100			0	7	(110)		

column gives the values published by Mulders * (Mu), Righini † (Ri), Dahme ‡ (Da), and Minnaert and Genard § (MG). For completeness the table contains all strong solar lines whether measured or not, and all Mulders' values are included. More doubtful observations are in parenthesis.

The Rowland Intensity Scale.—The data in Table I enable one to obtain a more continuous evaluation of the Rowland intensity scale in the infra-red than that published by Mulders (*loc. cit.*). The intensity numbers given in the revised Rowland, which are the only estimates hitherto available, are evaluated in Table II, where each tabulated value is a mean of at least three lines. As the table shows there is a severe discontinuity in the neighbourhood of 7200 A., and hence the intensity numbers beyond this wave-length are not at all comparable with the visible spectrum. The new Rowland numbers to be published by Mr. Babcock and Miss Moore have avoided this discontinuity, and agree in the mean with the red end of the visible spectrum. The evaluation of the new scale is given in Table III, at the bottom of which is included for comparison the old Rowland scale from 5800–6600 A.

* G. F. Mulders, *loc. cit.* † G. Righini, *Zeit. f. Astrophys.*, **10**, 344, 1935.
‡ A. Dahme, *Zeit. f. Astrophys.*, **11**, 93, 1935.
§ M. Minnaert and J. Genard, *Zeit. f. Astrophys.*, **10**, 377, 1935.

TABLE II

The Equivalent Breadth in Milli-angstroms represented by Intensities in the "Revised Rowland"

Wave-length Region	Intensities quoted in Revised Rowland						
	−1	0	1	2	3	4	5
6600–7000	11	18	35	55	73	82	117
7000–7200	29	31	51	57	117	103	...
7200–7600	48	55	76	120	...	...	...
7600–8000	...	43	56	90	110	...	...
8000–8400	36	59	78	184	165	...	...
8400–8900	36	68	106	153	...	...	...
Visible spectrum 4000–6600	11	20	37	56	76	103	128

TABLE III

The Equivalent Breadth in Milli-angstroms represented by the New Intensity Estimates of Babcock and Moore

Wave-length Region	New Intensity Estimates of Babcock and Moore									
	0	1	2	3	4	5	6	7	8	9
6600–7000	17	19	33	47	58	81	...	...	...	...
7000–7200	...	27	30	31	48	...	64	58	...	...
7200–7600	...	...	...	44	65	60	...	70	90	100
7600–8000	...	...	45	...	45	65	58	85	...	140
8000–8400	...	27	38	72	85	115	...	...	...	...
8400–8900	...	26	30	48	60	77	118	130	...	170
Mean 6600–8900	16	23	35	46	68	85	100	100	120	140
Old or Revised Rowland Scale 5800–6600	16	26	40	55	66	76	92	123	150	

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ON THE INTERNAL CONSTITUTION OF ECLIPSING BINARIES.

Zdeněk Kopal.

Until quite recently the study of the internal constitution of stars was a subject of purely theoretical research. Later, however, the possibility has been pointed out of obtaining some indications concerning the internal constitution of stars from certain observable phenomena in case of close binaries. These phenomena are :

1. The advance of periastron in orbits of close binaries (Russell *).
2. The ellipticity of the components (Walter †).
3. The librations of components (Walter ‡ and Thürring §).

The studies of these phenomena led to the first empirical proofs of all theoretical considerations as to the density distribution in the interiors of stars, and indeed Russell's first results brought some surprise. The observations of Y Cygni seemed to indicate only a moderate density concentration $\left(\frac{\rho_c}{\rho_m} \sim 6\right)$ in striking disagreement with the widespread belief that the stars follow the polytrope model $n = 3$; the re-determination of elements based on the latest material available || led only to an increase of $\frac{\rho_c}{\rho_m} \sim 16$. Russell's results have been criticised several times from the practical as well as from the theoretical point of view ¶ and cannot be regarded as final ; but a study of other phenomena seems to lead to the same conclusions. Walter † showed in 1931 that the ellipticities of the components in β Lyræ type systems indicated an average density concentration $\frac{\rho_c}{\rho_m} \sim 2$, and a year later he arrived at similar results by discussing the librations in some special systems (β Lyræ and W Ursæ Majoris).‡ Thus all kinds of observational evidence seem to be in striking contradiction with previous theoretical considerations.

The aim of the present paper is to throw more light on the questions related with the problem with which we are concerned here. The observational material available, which has increased considerably since the time when the above-mentioned investigations were carried out, will give us a sure basis for advancing our conclusions as far as the present state of research permits.

The possibilities quoted in 1 and 2 are applicable only in special cases and also the theory of these phenomena remains open to serious objections.

* *M.N.*, **88**, 641, 1928. † *Königsb. Veröff.*, **2**, 1931.
‡ *Königsb. Veröff.*, **3**, 1932. § *A.N.*, **253**, 73, 1934. || *M.N.*, **91**, 212, 1930.
¶ *Cf.* H. Klaŭder, "Über den inneren Aufbau von Bedekungsveränderlichen," *A.N.*, **255**, 1, 1935.

An excellent opportunity, however, is offered us by the ellipticities of stars because of the considerable observational material and also because of the profound theoretical analysis of the underlying physical problem by S. Chandrasekhar.*

This method was first followed, so far as I know, by K. Walter.† Two years before Chandrasekhar's papers, Walter dealt with the same problem and got the results which anticipated the later and more complete investigations of Chandrasekhar. For the ellipticity Walter deduced the expression :

$$\epsilon^2 = K\frac{\mathfrak{M}_j}{\mathfrak{M}_i}\left(\frac{r_i}{R}\right)^3, \tag{1}$$

where $\frac{\mathfrak{M}_j}{\mathfrak{M}_i}$ is the mass-ratio, $\left(\frac{r}{R}\right)$ the radius of the distorted body expressed in terms of the orbital radius as unit and K a constant specifying the density concentration. Walter supposed the internal constitution of all stars to be the same, and using the material compiled in two tables he deduced K for each system separately. These individual values differed widely—about one-half of them being outside possible limits ‡—but their mean value indicated a density concentration only $\frac{\rho_c}{\rho_m} \sim 2$, which corresponds nearly to the polytrope $n = 1$.

The uncertainty of Walter's results was partly due to the scarcity and, in some cases, also to the rather unfortunate choice of material, but the chief error Walter committed was putting for r the major semi-axis of the spheroidal configuration instead of the radius of the configuration in its undistorted state. As $\left(\frac{r}{R}\right)$ occurs in high powers, the error was considerable and was apt profoundly to jeopardise his results.

The ellipticities of components in a double star system are due both to the rotational and tidal forces which the components exert upon one another. It follows from Chandrasekhar's theory that the oblateness $e\left(=\frac{a-b}{a}\right)$ in the meridional plane with which we are concerned, to the fifth order in ν is

$$e = \frac{3}{2}\cdot\frac{\mathfrak{M}_j}{\mathfrak{M}_i}\nu_i^3[\Delta_2 + \tfrac{5}{12}\Delta_4\nu_i^2], \tag{2}$$

where ν is the radius of the star in its undistorted shape expressed in terms of the orbital radius as unit and Δ_2 or Δ_4 are constants specifying the constitution of the star. As can be seen, Walter's solution (1) is true so far as the terms in ν^5 and higher order terms remain inappreciable.

Both components, however, are generally of different forms and only the mean ellipticity can be taken from observations. But as Walter showed,§

* *M.N.*, **93**, 390, 449, 462, 539, 1933.
† *Königsb. Veröff.*, **2**, 59, 1931.
‡ See Klaŭder's paper quoted sub. ¶, p. 4.
§ *Königsb. Veroff.*, **2**, 63, 1931.

the mean ellipticity ϵ_{ij}^2 can—with a very high degree of accuracy—be written

$$\epsilon_{ij}^2 = L_i\epsilon_i^2 + L_j\epsilon_j^2 + \tfrac{3}{4}L_iL_j(\epsilon_i^2 - \epsilon_j^2)^2, \tag{3}$$

where L_i and L_j are the luminosities of the components (the luminosity of the system is taken as unit). The quadratic term on the right side of (3) plays an unimportant rôle and can generally be neglected.

For small ellipticities we may write $\epsilon^2 = 2e$ and inserting (2) in (3) we get

$$\epsilon_{ij}^2 = 3\Delta_2\left[L_i\frac{\mathfrak{M}_j}{\mathfrak{M}_i}\nu_i^3 + L_j\frac{\mathfrak{M}_i}{\mathfrak{M}_j}\nu_j^3\right] + \tfrac{5}{4}\Delta_4\left[L_i\frac{\mathfrak{M}_j}{\mathfrak{M}_i}\nu_i^5 + L_j\frac{\mathfrak{M}_i}{\mathfrak{M}_j}\nu_j^5\right], \tag{4}$$

but already for $\epsilon^2 > 0{\cdot}01$ the inequality $2e > \epsilon^2$ must be taken into account.

The constants Δ_2 and Δ_4 are characteristic of the internal constitution of both components and their values as functions of the polytrope class were tabulated by Chandrasekhar.* The higher the index of the coefficient, the less sensitive it is to the variation of the polytrope class, the coefficients with high indices being nearly equal to unity. We can then specify the internal constitution of a star by means of the first coefficient alone (setting $\Delta_4 = 1$),

$$\Delta = \frac{5}{12}\cdot\frac{\epsilon_{ij}^2 - L_i\frac{\mathfrak{M}_j}{\mathfrak{M}_i}\nu_i^5 - L_j\frac{\mathfrak{M}_i}{\mathfrak{M}_j}\nu_j^5}{L_i\frac{\mathfrak{M}_j}{\mathfrak{M}_i}\nu_i^3 + L_j\frac{\mathfrak{M}_i}{\mathfrak{M}_j}\nu_j^3}, \tag{5}$$

and proceed further by approximations.†

So far we have tacitly assumed the constitution of both components to be the same; indeed if we do not do so, (4) is not justified and no further progress in this way can be made. Because this assumption does not seem to us to be evident ‡ and we wish to examine its validity in what follows, we shall divide our material into groups containing stars of the same spectral class and reject all systems where both components are considerably different in spectrum or mass.

The table on p. 861 contains all elliptical binary systems of whose elements we have a reliable knowledge. The individual columns give: (1) the name, (2)–(3) the spectra of the components, (4) the luminosity $L_i (L_j = 1 - L_i)$, (5) the mass-ratio $\frac{\mathfrak{M}_j}{\mathfrak{M}_i}$, (6) ν_i, (7) ν_j, (8) the radii-ratio k, (9) the observed ellipticity ϵ_{ij}^2, (10) Δ, (11) "uniform" (U) or "darkened" (D) solution adopted, (12) reference.

The spectra are taken from Prager's *Katalog und Ephemeriden der veränderlichen Sterne für 1936*, § the brackets indicate hypothetical spectra

* *M.N.*, 93, 456, 1933.

† It is to be pointed out that the symbol Δ defined by (5) does not mean the same as Δ_2.

‡ In Walter's investigation systems with very unequal components form an isolated group, differing entirely from others.

§ *Kleinere Veröff. Berlin Babelsberg*, Nr. 15, 1936.

which were determined from the ratio of surface brightnesses. The mass-ratios were either taken from spectroscopic orbits or they were determined by the writer's method* from the elements, the spectrum and the mass-luminosity law (the latter are marked by an asterisk). The values ν_i and ν_j were deduced from r_i, r_j and ϵ_{ij}^2, supposing both bodies to be prolate spheroids. The ellipticities were corrected as regards the reflection effect according to the theory of Eddington,† where the reflection constants have been available. Systems with $\epsilon_{ij}^2 < 0{\cdot}01$ were not taken into account.

The theoretical values of Δ_2 for different polytrope models were computed by Chandrasekhar ‡ and may be noted here :

	$n=0$	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
Δ_2	2·500	1·520	1·148	1·029	1·003	1·000

B. Thürring has recently pointed out that owing to the neglect of terms of the order two and higher in $v\left(=\frac{\omega^2}{2\pi G\rho}\right)$ in the solution of the fundamental equation of the rotational problem Chandrasekhar's ellipticities are too small by about 10 per cent. Therefore, in order to get results approaching reality as near as possible we have to increase Δ_2 by nearly the same amount to get constants Δ defined by (5). Indeed the observational material itself indicates that the upper limit of Δ will lie between 2·7 and 2·8—that is, by about 10 per cent. less than that computed by Chandrasekhar.

Now let us consider more fully on what Δ depends—the constant characterising stellar density concentration.

Fig. 1 shows the relation which seems to exist between Δ and the radii-ratio, for each spectral class separately.

It is evident that in all groups Δ decreases with increasing k. In a previous paper the writer has shown § that k can be regarded as an indication of the interval since fission. Our present statement means then that the density concentration depends on the age of the system : the older the binary, the more evident the density concentration. Our diagrams enable us, however, to pursue the present problem still further. We see that the relationships shown in fig. 1 are by no means linear ; there are only slight variations in Δ_2 for $k > 1$, and with decreasing k the density concentration begins to increase in a very striking way.

The systems with $k > 1$ come very nearly to their upper possible limit in Δ, *i.e.* the observational evidence indicates that at the moment of fission stars come closely to the Maclaurin model—that they are homogeneous or nearly homogeneous. This explains, by the way, the fact that in every case the result of fission is two components with nearly equal masses, which is characteristic of the Maclaurin model. In the case of an unhomogeneous body with density increasing inwards quite another feature should be expected. As time passes, there is a tendency to increase the central density and in later stages of binary star evolution we have to deal with stars showing a fairly high degree of density concentration.

* *Zeitschrift für Astrophysik*, **9**, 239, 1935.
† *M.N.*, **86**, 320, 1926.
‡ *M.N.*, **93**, 756, 1933.
§ *Königsb. Veröff.*, **2**, 63, 1931.

The second question we want to settle is whether the density concentration in stars depends upon their position in the Hertzsprung-Russell

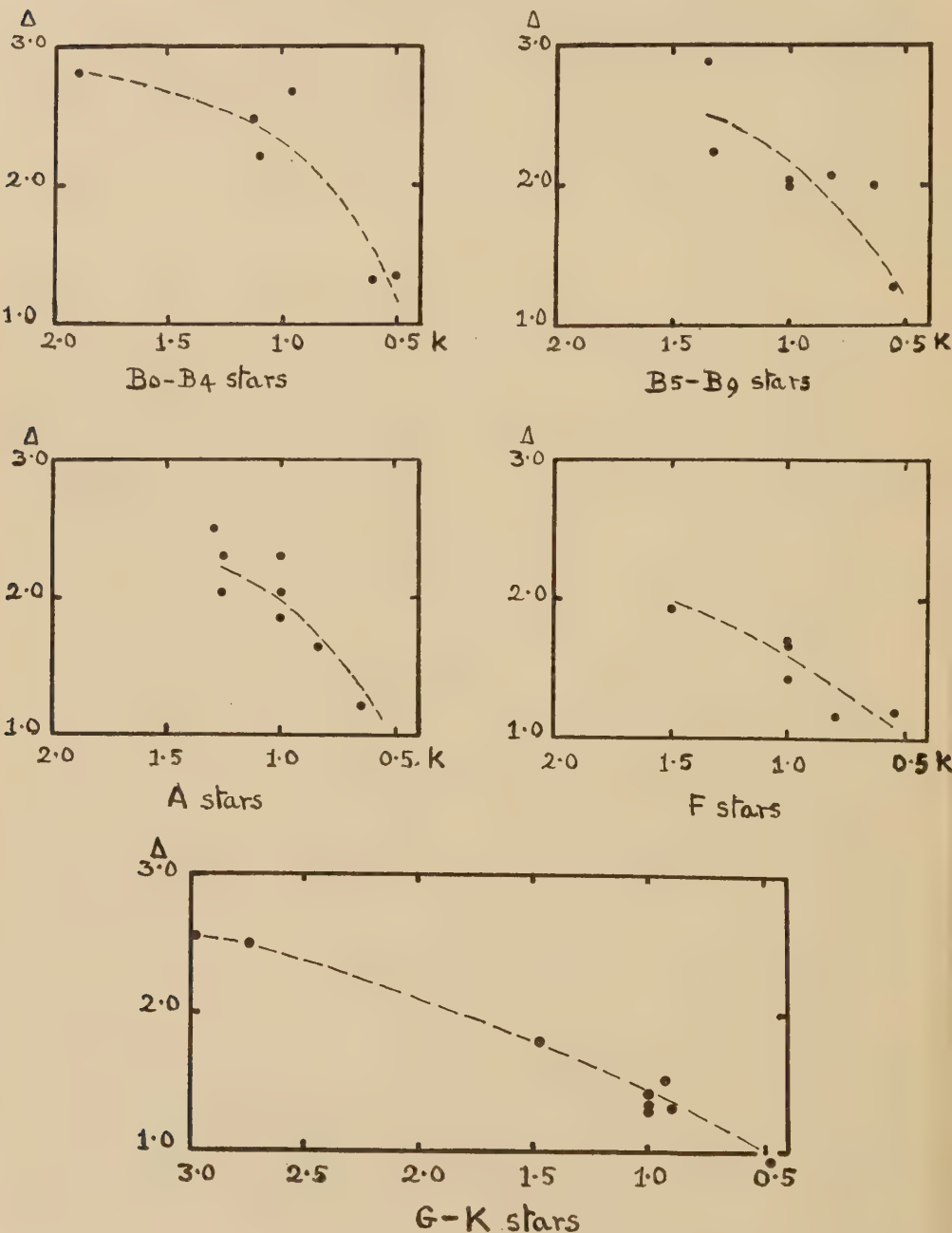

Fig. 1.

diagram. Owing to the dependence on k we have, of course, to compare only the stars with nearly the same value of radii-ratio.

Fig. 2 shows the relation between Δ and the spectral class for stars with $1{\cdot}1 > k > 0{\cdot}9$.

It can clearly be seen that Δ is decreasing along the series $B \rightarrow A \rightarrow K$, *i.e.* that the density concentration tends to increase with advancing spectral type. This is, of course, valid only for stars of the main sequence, to which

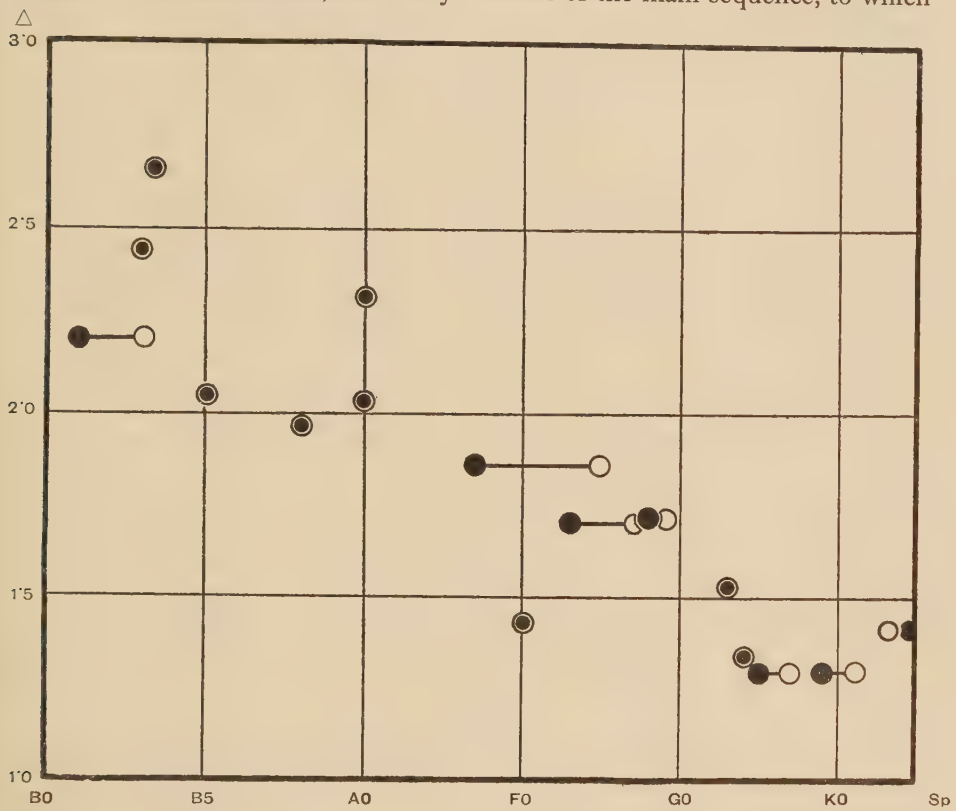

FIG. 2.

the majority of the systems discussed by us belong. The only giants are W Crucis and SS Camelopardalis. Their characteristics seem to indicate that they are both homogeneous or nearly homogeneous, but we must be careful to confine this to all giants. Both systems, indeed, are according to their radii-ratios at the beginning of their life as binaries and possibly their present state is a consequence of their age rather than of their position in the Hertzsprung-Russell diagram.

The dependence of the density concentration upon the spectral type may also be of fundamental physical significance. If the stellar energy originates in formation of heavy elements out of simpler ones, the number of heavy atoms must increase as the star grows old and they may well sink towards the star's centre. Although other influences may interfere, the above statement, if confirmed by future investigation, will be a contribution in support of the view that transmutation of elements really occurs in stellar interiors.

All the above statements are based on the material taken from eclipsing binaries and the question is whether these can be generalised with respect to all "ordinary" stars. Unfortunately there exists no possibility of proving it for simple stars and there are many reasons in favour of the view that binaries are rather exceptional phenomena as to their internal constitution. Anyhow, the best representations, *i.e.* representations coming as near as possible to simple stars, are components of a system in a late stage of evolution, with k say $>>1$. Looking at fig. 1 we see, however, that for such stars Δ rapidly reaches small values which correspond to a much more pronounced density concentration. Unfortunately the stronger the density concentration, the less appreciable are changes in Δ and, moreover, systems in later stages of their evolution are only slightly oblate or not at all. Consequently our present investigation can hardly throw more light on the constitution of simple stars. Possibly they follow the models with a high density concentration, such as polytrope $n=3$, and the moderate density concentration suggested recently from several sources may be characteristic only for binaries after fission and in early stages of their evolution. Fig. 1 even indicates this; nevertheless we have but little chance of obtaining fuller knowledge.

Owing to the cosmogonic importance of the present results all possible circumstances which might interfere should be carefully considered. The theoretical basis of our investigation is the theory of Chandrasekhar which cannot, in some regards, be considered as corresponding completely to reality.

Thus the disturbing component was supposed to be a mass point, but this does not probably affect our results at all. Apart from the fact that the effect of the secondary, if considered as an ellipsoid, will become appreciable only in terms of the order 6 and higher, for the majority of close binaries only one (the less massive) body has become an ellipsoid, while the departure from spherical form of its more massive companion is usually far less appreciable, so that the latter can be taken with a very high degree of accuracy as a mass point (or sphere, which is equivalent, whatever its density distribution may be).

The solution of the fundamental differential equation in the rotational problem was supposed by Chandrasekhar (according to von Zeipel) to be an expansion in v where all members of the order 2 and higher can be neglected. This has already been found not to be justified from the theoretical point of view as well as from observation (see above).

A more serious defect of the theory is that it has considered the distorted configurations to rotate with a constant angular velocity, which can hardly be true. Furthermore, Chandrasekhar's investigations have dealt with the polytrope configurations only, and it is quite possible that the density distributions in interiors of stars have such special features that it may not be safe to draw general conclusions from the study of polytropes alone. Since our knowledge of these points is not complete, it is possible that our results will need some modification—although it seems unlikely.

One final remark may be added. It was already sufficiently noted above

TABLE

(*For the explanation of the columns, see p.* 856)

Bo–B4 stars											
TT Aur	B3	B3	0·660	0·79	0·326	0·290	1·12	0·259	2·452	U	Joy-Sitterly, *Ap. J.*, **73**, 77.
RZ Cen	B2	[B6]	0·740	0·51	0·228	0·457	0·50	0·19	1·363	U	Shapley, *Princ. Contr.*, **3**.
U CrB	B3	B3	0·938	0·38	0·166	0·277	0·60	0·020	1·288	D	Baker, *Laws. Bull.*, **29**.
β Lyr	B2	B8	0·553	0·38	0·497	0·250	1·89	0·391	2·823	U	Guthnick, *Berl. Akad. Ber.*, **5**, 222.
V Pup	B1	B3	0·600	0·77	0·390	0·358	1·09	0·384	2·190	U	Shapley, *Princ. Contr.*, **3**.
Z Vul	B3	B3	0·749	0·46	0·266	0·280	0·95	0·154	2·677	D	Shapley, *Princ. Contr.*, **3**.
B5–B9 stars											
δ Aql	B8	B8	0·572	0·83	0·236	0·236	1·00	0·078	1·970	U	Wylie, *Ap. J.*, **56**, 232.
SV Cen	B8	[A6]	0·844	0·46*	0·382	0·281	1·35	0·287	2·884	D	Dawson, *La Plata*, **6**, 3.
μ Her	B5	B5	0·658	0·63	0·274	0·334	0·82	0·191	2·058	U	Pavel, *A.N.*, **217**, 177.
U Oph	B5	B5	0·535	0·70	0·244	0·244	1·00	0·05	2·040	D	Shapley, *Princ. Contr.*, **3**.
V Ser	B8	[A5]	0·587	0·69*	0·175	0·286	0·63	0·097	1·980	U	Shapley, *Princ. Contr.*, **3**.
λ Tau	B3	[A4]	0·796	0·20	0·289	0·216	1·33	0·098	2·231	D	Stebbins, *Ap. J.*, **51**, 193.
RS Vul	B8	B8	0·804	0·31	0·201	0·262	0·76	0·052	1·252	U	Dugan, *Princ. Contr.*, **3**.
A stars											
AN And	A7	[F5]	0·732	0·52	0·248	0·248	1·00	0·078	1·850	U	Huffer, *Washb. Publ.*, **15**, 117.
S Ant	A8	A8	0·670	0·74	0·414	0·321	1·30	0·438	2·503	U	Joy, *Ap. J.*, **64**, 287.
SX Aur	A3	[A3]	0·586	1·00*	0·305	0·359	0·84	0·190	1·649	U	Gaposchkin, *Berl. Bab. Veröff.*, **9**, 5.
TV Cas	A0	A0	0·564	0·59	0·265	0·265	1·00	0·078	2·031	D	Shapley, *Princ. Contr.*, **3**.
SZ Cen	A2	[A3]	0·590	0·37*	0·288	0·231	1·25	0·135	2·295	U	Shapley, *Princ. Contr.*, **3**.
TW Dra	A5	[G4]	0·894	0·37	0·207	0·348	0·65	0·059	1·264	U	Baker, *Laws. Bull.*, **33**.
RX Her	A0	A0	0·500	0·90	0·203	0·203	1·00	0·060	2·320	D	Shapley, *Princ. Contr.*, **3**.
V Tri	A3	[F2]	0·863	0·70*	0·346	0·277	1·25	0·226	2·033	U	Gaposchkin, *Berl. Bab. Veröff.*, **9**, 5.
F stars											
RS CVn	F4	G8	0·600	0·93	0·311	0·389	0·80	0·154	1·116	U	Sitterly, *Ap. J.*, **53**, 69.
RV CVn	F8	[F9]	0·575	1·00	0·313	0·313	1·00	0·190	1·708	D	Gaposchkin, *Berl. Bab. Veröff.*, **9**, 5.
U Peg	F3	[F7]	0·603	0·71	0·410	0·410	1·00	0·407	1·680	D	Shapley, *Princ. Contr.*, **3**.
AB Per	F0	[F2]	0·550	1·00*	0·193	0·346	0·55	0·078	1·199	U	Gaposchkin, *Berl. Bab. Veröff.*, **9**, 5.
TY Pup	F0	[F0]	0·500	0·68*	0·269	0·269	1·00	0·190	1·472	U	Gaposchkin, *Berl. Bab. Veröff.*, **9**, 5.
W UMa	F8	F8	0·690	0·72	0·472	0·315	1·50	0·453	1·936	D	Walter, *Königsb. Beob.*, **45**, 1.
G–K stars											
AB And	G5	[G7]	0·520	0·20*	0·344	0·344	1·00	0·452	1·301	U	Gaposchkin, *Berl. Bab. Veröff.*, **9**, 5.
SS Cam	G1	F5	0·640	0·40*	0·419	0·141	2·97	0·172	2·537	U	McDiarmid, *Pop. Astr.*, **24**, 668.
RZ Cnc	K0	K4	0·712	0·51	0·222	0·464	0·48	0·190	0·939	D	Gaposchkin, *Berl. Bab. Veröff.*, **9**, 5.
RW Com	G4	[G4]	0·550	0·95*	0·353	0·535	0·90	0·294	1·347	D	Gaposchkin, *Berl. Bab. Veröff.*, **9**, 5.
W Cru	G0	[F8]	0·676	0·45*	0·469	0·173	2·74	0·297	2·471	U	Shapley, *Princ. Contr.*, **3**.
RT Lac	G9	K1	0·602	0·82	0·289	0·289	1·00	0·098	1·306	D	Shapley, *Princ. Contr.*, **3**.
SW Lac	G3	G3	0·541	0·57	0·367	0·395	0·93	0·344	1·541	U	Schilt, *B.A.N.*, **68**.
U Sct	G5	[K3]	0·900	0·52*	0·390	0·265	1·47	0·183	1·806	D	Shapley, *Princ. Contr.*, **3**.
RW Dor	K5	[K7]	0·668	0·14*	0·286	0·286	1·00	0·277	1·442	U	Gaposchkin, *Berl. Bab. Veröff.*, **9**, 5.

that the ellipticity of components and of the system as a whole depends on ν, the mass-ratio and the internal constitution of stars. In the present paper we have tried to examine the dependence of Δ on the radii-ratio and spectrum. Now *vice versa*: if we know ν and k, which can both be taken from the light curve and the spectrum of the primary (that of the secondary can be approximately determined from the ratio of surface brightnesses), we can take from fig. 1 the probable value Δ corresponding to known k and spectrum and determine in this way the mass-ratio of the system and so the absolute dimensions, masses and densities without a knowledge of spectroscopic orbit.

Conclusion.—The underlying physical problem in the present paper was to obtain some general idea regarding the density concentration in the interiors of stars, using eclipsing binaries as material. Our investigation has led to the following conclusions :—

1. The density concentration depends on the age of the system. The components of a new-born binary are homogeneous or nearly homogeneous.
2. As time elapses, the density concentration begins to become appreciable, and at the end of the evolution of a system both components come close to the model requiring a high density concentration.
3. The stars of the main sequence show an increase of the density concentration with advancing spectral type.

All that has been said may be generalised for spectroscopic binaries.

1423, *u Pernikářky*,
Praha XVI :
1936 *April.*

THE APPARENT STATISTICAL RELATION BETWEEN PERIOD AND ECCENTRICITY IN VISUAL BINARY ORBITS.

W. S. Finsen.

It is well known that among orbits of visual double stars there is an apparent tendency for large values of the eccentricity to be associated with long periods. In his recent second edition of *The Binary Stars*, p. 208, Aitken quotes the following table in support :—

Number of Stars	$\bar{P}$	$\bar{e}$
	y	
13	11·69	0·455
13	24·11	0·489
13	36·89	0·383
13	49·33	0·529
13	77·43	0·519
13	98·44	0·468
13	126·91	0·601
13	186·18	0·579
12	308·82	0·627

Some attempts have been made in recent years to explain this relationship on physical grounds, on the assumption that it is real. The question of its reality has been dealt with previously by Seyfert * and Barbier.† Seyfert found that selections depending on the chance of discovery of a pair had very little effect on the eccentricity distribution for different period classes, and concluded that the observed increase of eccentricity with period was apparently a real effect. On the other hand, as Barbier remarked, such discovery selection is probably of less importance than computational selection, for experience shows that the chance of computing an orbit depends greatly on some of the elements. Barbier showed, in addition, that the relation had decreased with the increase in number of available orbits.

There are several possible sources of computational selection. One of these is particularly likely to lead to an illusory relation between period and eccentricity: when the period is long, the requirement of a well-defined apparent ellipse favours the cases where the eccentricity is large and the date of periastron lies within the interval for which observations are available. This would give rise to an illusory positive correlation between period and eccentricity. If we substitute for the element T the mean anomaly M on some fixed date near the mean date of double star measures in general (*e.g.* 1900), we should in addition expect to find an illusory negative correlation between P and M. The quantitative estimation of such selection effects is attended with difficulty and uncertainty.

The present investigation is based on the catalogue of double star orbits compiled in 1933.‡ The orbits of OΣ 98, ζ Cnc $A+B$, $C+c$, σ_2 UMa, Σ 1639, 61 Cyg, with periods in excess of 500 years, were excluded as too unreliable. The systems Capella, Mizar, ζ Cnc $C+c$, C, ξ UMa $A+a$, A, were also omitted. The remaining orbits numbered 135.

The distribution of P and e is shown in the table.

The table is very evidently affected by sampling errors of a degree

P, years / e	0–50	50–100	100–150	150–200	200–300	300–500	Sums
0·00 – 0·10	0	0	0	0	0	0	0
0·10 – 0·20	6	1	3	1	1	1	13
0·20 – 0·30	8	1	2	0	0	0	11
0·30 – 0·40	7	4	3	2	0	0	16
0·40 – 0·50	9	5	5	3	3	2	27
0·50 – 0·60	7	6	5	2	6	1	27
0·60 – 0·70	3	3	1	3	1	2	13
0·70 – 0·80	4	2	5	0	2	0	13
0·80 – 0·90	2	2	0	1	2	0	7
0·90 – 1·00	1	2	2	1	1	1	8
Sums	47	26	26	13	16	7	135

* *Harvard Bulletin*, 896, 1934. † *C.R.*, **199**, 930, 1934. ‡ *U.O.C.*, **91**, 1934.

sufficiently pronounced to cause distrust in the weak association of e with P. This is further borne out by the correlation coefficient which may be derived :

$$r_{Pe} = +0{\cdot}18 \pm 0{\cdot}08 \text{ (standard error).}$$

This small correlation coefficient, if real at all, indicates only a very weak causation linkage between the variables. On the other hand, the size of its standard error and the possibility of selection effects cast doubt on its reality. This question will now be examined more fully.

Apart from sampling fluctuations and selection effects, the total correlation coefficient r_{Pe} between P and e may be affected by correlations of each with neglected elements. It is therefore desirable to compute the partial correlation coefficient between P and e, i.e. $r_{Pe\,.\,ai\omega\Omega M}$, the other five elements having assigned values. As mentioned earlier, the element T is replaced by M, the mean anomaly at 1900. If in addition the partial correlation coefficient for every other pair of elements is computed, the result may yield valuable information regarding sampling and selection errors.

Omitting OΣ 4 and Σ 2438, which have zero inclination and consequently lack Ω and ω, the number of orbits available for this purpose was 133. Taking Ω and M in the range 0° to 180°, and i and ω between 0° and 90°, the mean values of the elements and the regression equations (referred to the means) are :

$\bar{P}$	112·7 years
$\bar{e}$	0·5134
$\bar{a}$	1″·348
$\bar{i}$	53°·62
$\bar{\omega}$	43°·83
$\bar{\Omega}$	85°·21
$\bar{M}$	78°·49

$$P = +82{\cdot}0\,e + 14{\cdot}9\,a - 0{\cdot}238\,i + 0{\cdot}0970\,\omega + 0{\cdot}190\,\Omega - 0{\cdot}460\,M$$
$$e = +0{\cdot}000407\,P - 0{\cdot}00654\,a - 0{\cdot}0000895\,i + 0{\cdot}000145\,\omega - 0{\cdot}00112\,\Omega - 0{\cdot}000283$$
$$a = +0{\cdot}00793\,P - 0{\cdot}698\,e + 0{\cdot}00674\,i + 0{\cdot}00366\,\omega - 0{\cdot}00200\,\Omega + 0{\cdot}00384\,M$$
$$i = -0{\cdot}00927\,P - 0{\cdot}702\,e + 0{\cdot}489\,a - 0{\cdot}0140\,\omega - 0{\cdot}0349\,\Omega - 0{\cdot}0294\,M$$
$$\omega = +0{\cdot}00745\,P + 2{\cdot}29\,e + 0{\cdot}526\,a - 0{\cdot}0278\,i - 0{\cdot}0838\,\Omega + 0{\cdot}0479\,M$$
$$\Omega = +0{\cdot}0503\,P - 59{\cdot}7\,e - 0{\cdot}994\,a - 0{\cdot}241\,i - 0{\cdot}289\,\omega - 0{\cdot}232\,M$$
$$M = -0{\cdot}137\,P - 17{\cdot}1\,e + 2{\cdot}15\,a - 0{\cdot}227\,i + 0{\cdot}186\,\omega - 0{\cdot}261\,\Omega$$

The following table shows the resulting partial correlation coefficients between pairs of elements, the remaining elements having assigned values :—

	P	e	a	M	i	ω
e	+·18					
a	+·34	−·07				
M	−·25	−·07	+·09			
i	−·05	−·01	+·06	−·08		
ω	+·03	+·02	+·04	+·09	−·02	
Ω	+·10	−·26	−·04	−·25	−·09	−·16

The standard errors of these coefficients range from ± ·077 for the largest to ± ·087 for the smallest.

This set of correlation coefficients presents some striking features. The largest, that for P and a, was to be expected and calls for no comment. The correlations found for Ω and M and for Ω and e are more surprising. It is difficult to imagine a real association of Ω with any other element; it is equally difficult to see how such associations could result from selection. These associations must therefore be ascribed to sampling fluctuations. If the 133 orbits are grouped according to Ω and the mean values of M and e formed for each group, these associations are easily seen :

Ω	$\bar{M}$	e
° °	°	
0 – 20	101	0·65
20 – 40	102	0·54
40 – 60	88	0·51
60 – 80	66	0·51
80 – 100	61	0·54
100 – 120	76	0·54
120 – 140	85	0·46
140 – 160	73	0·41
160 – 180	40	0·45

It will be noticed that the accidental $\Omega - e$ and $\Omega - M$ relations are of quite as pronounced a character as the $P - e$ relation for which physical reasons have been sought.

Real correlations between M and the other elements are also difficult to imagine, but in this case it is likely that selections of the type referred to earlier may give rise to illusory correlations. In fact, the negative correlation actually found for P and M is in the sense to be expected, with which the positive correlation found for P and e is in agreement. The correlations found for Ω and M and for Ω and e indicate, however, that sampling errors are quite adequate explanation. Indeed, many unsuccessful attempts to assess and remove effects of selection led only to the conclusion that they were of the same order of size as the sampling errors.

It seems that no significance should be attached to the period-eccentricity relation in visual double star orbits : it is well within the range of error due to sampling fluctuations and, possibly, selection effects. It may be, of course, that a statistical relation between these two elements does in reality exist ; but if so, it is almost certainly very small and not yet discoverable.

The spectroscopic binaries also exhibit an association of long period with large eccentricity, while the average eccentricity is much smaller than for visual binaries. If the spectroscopic and visual binaries are combined there is no striking discontinuity in the apparent period-eccentricity curve, and this would seem to lend support to the reality of the relation in both classes. Against this it may be objected that it is not at all certain that we may legitimately assume that the spectroscopic and visual binaries represent different stages in the same evolutionary sequence. Hence a *real* period-eccentricity relation in the spectroscopic binaries does not necessarily confirm a similar *suspected* relation in the visual binaries. Furthermore, in the case of the former the reality of the relation is also open to doubt, for the effects of

discovery selection are likely to be serious. The discovery of a spectroscopic binary results from discordant radial velocity determinations or doubling of lines, and the chance of discovery depends therefore, apart from other considerations, on the shape and amplitude of the velocity curve. This approximates to a sine curve for small eccentricities, but in the case of large eccentricity it is characterised by long intervals of nearly constant velocity interrupted by sharp peaks, while the extreme velocity range is larger. Hence for short periods velocity determinations made at random are more likely to lead to discovery of a spectroscopic binary when the eccentricity is small. The case is different when the period is long. The amplitude of the velocity curve is now generally small and comparable with, or even within, the range of error, except for the higher eccentricities, so that while as before the shape of the curve for high eccentricity may be prejudicial to discovery, small eccentricities may be even more disfavoured. In this way an apparent period-eccentricity relation would result, the large number of short period binaries having in general small eccentricities, while the few long periods discovered tend to larger eccentricities.

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ON THE SYSTEMATIC DISTANCE-EFFECT IN VELOCITY ELLIPSOID DETERMINATIONS.

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1. With the progress of the theory of galactic rotation attention has been drawn to the fact that the direction of the vertex, as determined by the ellipsoidal distribution of stellar motions, shows a pronounced deviation from the direction of the galactic centre as determined from Oort's terms of galactic rotation. As I have shown in one of my earlier papers,* it may be assumed that the direction of the vertex is towards $l = 348°$, which is 23° away from $l = 325°$, the longitude of the galactic centre.

This discrepancy has given much concern to investigators, and in particular the author has tried to throw light on this problem by advancing a dynamical theory of stellar motions, more general than the rotational theory, a subject which quite recently has also been treated by Professor Milne † and P. Wellmann.‡ Incidentally in that paper I found that systematic group motions of stars of the nature of galactic rotation must influence the distribution of stellar velocities to an extent which is by no means negligible.§

* *Zeit. f. Aph.*, 4, 3, 1932. † *M.N.*, 95, 560, 1935. ‡ *Zeit. f. Aph.*, **10**, 1, 1935.

§ In the paper quoted I made even a still more far-reaching assumption, namely, that the ellipsoidal distribution of the early B stars, which formed the group of stars to which theoretical results were applied in the first line, is explained by this effect. This assumption was based upon the well-known fact that the velocity distribution of these stars is almost exactly spherical.

In a subsequent paper * I developed a theory which permits the determination of the magnitude of the above effect. As the paper is written in Russian and as the principal result in it will be used here, it may be useful to repeat briefly the arguments.

2. We assume first that the ellipsoid is determined according to the well-known Charlier method which in essentials consists in the determination of the second-order moments of the velocity distribution $N_{ijk}\,(i+j+k=2)$ whereby the squares of the axes of the ellipsoid σ_1, σ_2, σ_3 are determined as the roots of the Jacobi equation

$$G(t)=\begin{vmatrix} N_{200}-t & N_{110} & N_{101} \\ N_{110} & N_{020}-t & N_{011} \\ N_{101} & N_{011} & N_{002}-t \end{vmatrix}=0 \qquad (1)$$

while the direction cosines of the axes l, m, n are given by the equations

$$\frac{l}{G_{11}}=\frac{m}{G_{12}}=\frac{n}{G_{13}}=(G_{11}^2+G_{12}^2+G_{13}^2)^{-1/2}, \qquad (2)$$

where G_{11}, G_{12} and G_{13} denote the values of the sub-determinants of $G(t)$ corresponding to different roots of equation (1).

We now propose to compute the effect of stellar group motions upon the values of the moments N_{ijk}. We assume that within the elementary volume around the point with co-ordinates x, y, z, the average velocities of stars relatively to the adopted origin are continuous functions of the co-ordinates

$$\bar{u}=\bar{u}(x, y, z)\,; \qquad \bar{v}=\bar{v}(x, y, z)\,; \qquad \bar{w}=\bar{w}(x, y, z). \qquad (3)$$

The origin is taken at the instantaneous place of the Sun and at rest relatively to the totality of stars surrounding it. This of course means that the algebraic sums of the velocity components $\bar{u}$, $\bar{v}$, $\bar{w}$, extending over all stars contained within a certain finite volume of space around the Sun, are zero. If this volume is sufficiently small, we may develop functions (3) into power series in x, y, z and retain only terms of the first order. The equations (3) then will read

$$\begin{aligned} \bar{u} &= u_x.x+\ u_y.y+u_z.z, \\ \bar{v} &= v_x.x+v_y.y+v_z.z, \\ \bar{w} &= w_x.x+w_y.y+w_z.z, \end{aligned} \qquad (3')$$

where u_x, u_y, . . . w_z will be constant coefficients numerically equal to the partial derivatives of $\bar{u}$, $\bar{v}$, $\bar{w}$ at the point $x=y=z=0$.

It is important to keep in mind that the velocities $\bar{u}$, $\bar{v}$, $\bar{w}$ do not refer to individual stars but are average relative velocities of stars within an elementary volume with the centre at x, y, z or, as we have put it above, relative group motions of stars. As regards the individual stellar velocities, they may be distributed according to any statistical law whatever. For the following discussion we shall make, however, the assumption that the individual velocities are distributed around the mean velocity according to

* *R.A.J.*, 10, 3, 1933.

a law which may be approximately represented by an ellipsoidal function with coefficients independent of the co-ordinates.* Thus the only variable parameters depending upon x, y, z will be the mean velocity components $\bar{u}$, $\bar{v}$, $\bar{w}$ introduced above. Denoting the distribution function by $\phi(u, v, w\,;\, x, y, z)$ we shall have in general

$$\phi(u, v, w\,;\, x, y, z) = He^{-\frac{1}{2}f}, \tag{4}$$

where f is a quadratic function of the velocity components

$$u' = u - \bar{u}\,; \qquad v' = v - \bar{v}\,; \qquad w' = w - \bar{w},$$

$$f = a_{200}u'^2 + a_{020}v'^2 + a_{002}w'^2 + 2a_{011}v'w' + 2a_{101}w'u' + 2a_{110}u'v'$$

and

$$H = (2\pi)^{-3/2}\sqrt{A}, \tag{4'}$$

where A is the determinant of the coefficients a_{ijk},

$$A = |\, a_{ijk}\,|.$$

We shall call this kind of velocity distribution *streaming*. It is obvious that we have tacitly assumed the distribution to be stationary.

3. It is easy to see that galactic rotation, which is revealed by the Oort's double-wave terms in proper motions and radial velocities of stars, is a particular case of streaming. Indeed, let us take, first, a system of rectangular counter-clockwise co-ordinates X, Y, Z with the origin at the galactic centre, the Z axis being at right angles to the galactic plane and the two other axes being oriented arbitrarily. If then W denotes the velocity of rotation at distance $R = (X^2 + Y^2)^{1/2}$ from the axis of rotation, then the velocity components U and V along X and Y axes will be :

$$U = -W\,X/R\,; \qquad V = W\,Y/R,$$

the third component being equal to zero. The relative velocity of a point $(X + dX,\ Y + dY$ and an arbitrary $Z)$ with respect to a point $(X,\ Y,\ Z)$ will have the components

$$\Delta U = \left[\left(W/R - \frac{dW}{dR}\right)X^2/R^2 - W/R\right]dX + \left(W/R - \frac{dW}{dR}\right)XY/R^2\,dY$$

$$\Delta V = -\left(W/R - \frac{dW}{dR}\right)XY/R^2\,dX - \left[\left(W/R - \frac{dW}{dR}\right)Y^2/R^2 - W/R\right]dY.$$

If we assume the point $(X,\ Y,\ Z)$ to designate the instantaneous place of the Sun and turn the axes so that the Y axis should point to that place, then in the above formulæ we have to put $X = 0$, $Y = R$, and we shall have

$$\Delta U = -W/R\,dY$$

$$\Delta V = \frac{dW}{dR}dX.$$

* It is known that we might just as well have made a more general assumption, that the distribution function is of Charlier A-type, without changing the results.

We have now only to replace dX and dY respectively by $-x$ and $-y$ and we will then have $\bar{u}$ and $\bar{v}$ in our previous notation applied to this particular case

$$\bar{u} = \frac{W}{R} y\,; \qquad \bar{v} = -\frac{dW}{dR} x,$$

from which it follows that

$$u_y = \frac{dW}{dR}\,; \qquad u_x = u_z = 0,$$

$$v_x = -W/R\,; \qquad v_y = v_z = 0,$$

$$w_x = w_y = w_z = 0,$$

or, introducing Oort's coefficients A and B,

$$\bar{u} = (A - B)y\,; \qquad \bar{v} = (A + B)x.$$

Thus we see that galactic rotation leads really to a particular case of our general formulæ characterising what we call streaming.

4. We now proceed to the computation of corrections to the elements of the velocity ellipsoid due to streaming. We first remark that in order to obtain the true ellipsoid we should have to determine it from stars *belonging to the same velocity distribution*, i.e. *according to what has been said above*, contained in an infinitely small volume of space. The reason of this is that, although we have made the assumption that the form of the velocity distribution is independent of the co-ordinates, the group velocities, or the average ones, do depend upon them. Thus the velocity ellipsoids corresponding to two elementary volumes separated from each other by a finite distance, however identical in size and form, are, so to speak, shifted relatively to one another.

Now, what are we really doing in practice? We can never determine, of course, any ellipsoid from very near stars only. In trying to get sufficient material we combine together all stars down to a certain apparent magnitude, which means, for instance, that we are mixing together stars belonging to a great (strictly speaking, infinite) variety of distributions. Thus the "observed" distribution of velocities is no more the simple "true" ellipsoidal one, but a mixture of such ellipsoidal distributions. If we make the quite reasonable assumption that the space density of stars does not vary appreciably within the volume of space filled by the stars which we treat, and further that there is no systematic variations in the average distance of stars in different parts of the sky, then all elementary volumes will contribute to the integrated distribution in the same proportion, and the total volume may be assumed to be a sphere S of radius ρ. If we denote the observed velocity distribution by $\Phi(u, v, w)$, then

$$\Phi(u, v, w) = \frac{1}{4/3\,\pi\rho^3} \iiint_{(S)} \phi(u, v, w\,;\, x, y, z)\,dxdydz, \qquad (5)$$

where S at the base of the integrals means that the integration is taken over the interior of the sphere S, and $\phi(u, v, w\,;\, x, y, z)$ is the ellipsoidal distribution

function (4) introduced above. In practice ρ will never mean the greatest distance of the stars concerned, since, whatever the real space distribution of the stars may be, the number per unit volume of stars which are used in the determination of the ellipsoid will always thin away as we approach the limiting apparent magnitude. Thus ρ will mean rather the "equivalent" radius of a sphere which, if uniformly filled by stars, would give the same average distance $\bar{r}$ as actually observed. In other words, we may in practice take simply

$$\rho = 4/3\ \bar{r}.$$

We now have to compute the moments of the function Φ, which we may call the observed distribution function. We first observe that the moments of the first order of Φ are all equal to zero, which means that the presence of streaming leaves the origin immobile relatively to the totality of stars. This becomes, however, almost obvious if we remark that the group motions $\bar{u}$, $\bar{v}$, $\bar{w}$ are linear functions of x, y, z, and thus every elementary volume has its antipodal volume in the opposite direction, which cancels out the effect of the former. Now, an easy computation shows that each moment of the second order

$$N_{ijk} = \iiint_{-\infty}^{+\infty} u^i v^j w^k \Phi(u, v, w) du dv dw \qquad (i+j+k=2)$$

is equal to

$$\frac{A_{ijk}}{A} + \frac{3}{4\pi\rho^3} \iiint_{(S)} \bar{u}^i \bar{v}^j \bar{w}^k dx dy dz,$$

where $\bar{u}$, $\bar{v}$, $\bar{w}$ are our group motions. A is the determinant figuring in (4′) and A_{ijk} its sub-determinant.

But in the last expression the quantity

$$\frac{A_{ijk}}{A}$$

is exactly the value of the moment which we would have obtained if there were no streaming.* On the other hand, inserting in this expression the developments of $\bar{u}$, $\bar{v}$, $\bar{w}$ (3′) and performing the integration, we obtain

$$N_{ijk} = N_{ijk}^0 + \rho^2/5\ (u_x^i v_x^j w_x^k + u_y^i v_y^j w_y^k + u_z^i v_z^j w_z^k), \qquad (6)$$

where N_{ijk}^0 denotes the value of the "true" moment. Or, finally, denoting by $5\mu_{ijk}$ the expression in the brackets which actually determines the correction to the corresponding moment, we shall have

$$N_{ijk} = N_{ijk}^0 + \rho^2 \mu_{ijk}. \qquad (6')$$

5. The above formulæ solve our problem for the case of space velocities. The case when the velocity ellipsoid is determined from radial velocities alone may be treated, however, in a way very similar to the above one.

If we denote by α, β, γ the cosines of the angles which the line of sight forms with the axes, then the group motion in the line of sight $\bar{v}_r$ will be

$$\bar{v}_r = \alpha\bar{u} + \beta\bar{v} + \gamma\bar{w}. \qquad (7)$$

* *Memoirs of the University of California*, 7, 1928.

The observed distribution function of velocities will be

$$\Phi(v_r) = \frac{3\sigma_r}{\sqrt{2\pi}\sigma_r\rho^3}\int_0^\rho \exp\left\{-\frac{(v_r-\bar{v}_r)^2}{2{\sigma_r}^2}\right\} r^2 dr, \tag{8}$$

the integration being taken over a volume of a cone. Here σ_r stands for the "true" dispersion, which is defined by the relation

$$\sigma_r{}^2 = N_r{}^0 = N_{200}{}^0\alpha^2 + N_{020}{}^0\beta^2 + N_{002}{}^0\gamma^2 + 2N_{011}{}^0\beta\gamma + 2N_{101}{}^0\gamma\alpha + 2N_{110}\alpha\beta. \tag{9}$$

On the other hand, the "observed" second-order moment in the radial direction N_r will be defined by

$$N_r = \int_{-\infty}^{+\infty} v_r{}^2\Phi(v_r)dv_r,$$

or, performing the integration using the above expression for $\Phi(v_r)$,

$$N_r = \sigma_r{}^2 + 3/\rho^3\int_0^\rho \bar{v}_r{}^2 r^2 dr,$$

which, after substituting (7) and integrating, becomes

$$N_r = N_r{}^0 + 3/5\,\rho^2 S^2, \tag{10}$$

where for brevity we have put

$$S = \alpha^2 u_x + \beta^2 v_y + \gamma^2 w_z + \beta\gamma(v_z + w_y) + \gamma\alpha(w_x + u_z) + \alpha\beta(u_y + v_x).$$

According to the classical procedure we determine the values of the moments N_{ijk} ("observed") by solving by the least-square method the system of equations of condition

$$N_r = N_{200}\alpha^2 + N_{020}\beta^2 + N_{002}\gamma^2 + 2N_{011}\beta\gamma + 2N_{101}\gamma\alpha + 2N_{110}\alpha\beta.$$

Now, putting as before $N_{ijk} = N_{ijk}{}^0 + \rho^2\mu_{ijk}$ and using (9) and (10), this equation reduces to

$$\alpha^2\mu_{200} + \beta^2\mu_{020} + \gamma^2\mu_{002} + 2\beta\gamma\mu_{011} + 2\gamma\alpha\mu_{101} + 2\alpha\beta\mu_{110} = 3/5\,S^2. \tag{11}$$

If, according to Charlier's scheme,* we divide the sky into 48 equal areas and give to each area equal weight irrespective of the number of stars, then the least square solution is equivalent to reducing to minimum value the integral

$$T = \iint(\alpha^2\mu_{200} + \beta^2\mu_{020} + \gamma^2\mu_{002} + 2\beta\gamma\mu_{011} + 2\gamma\alpha\mu_{101} + 2\alpha\beta\mu_{110} - 3/5\,S^2)^2 ds \tag{12}$$

taken over the surface of the sphere of unit radius.

This gives after algebraic transformations the following equations for the determination of the μ_{ijk} :—

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* *Memoirs of the University of California*, 7, 1928.

$$\left.\begin{aligned}
\mu_{200}+\mu_{020}+\mu_{002} &= 3/25[(u_x^2+v_y^2+w_z^2)+(u_x+v_y)^2+(v_y+w_z)^2\\
&\quad +(w_z+u_x)^2+(u_y+v_x)^2+(v_z+w_y)^2+(w_x+u_z)^2]\\
\mu_{200}-\mu_{020} &= 3/35[4(u_x^2-v_y^2)+2(u_x-v_y)(u_x+v_y+w_z)\\
&\quad +(u_z+w_x)^2-(v_z+w_y)^2]\\
\mu_{200}-\mu_{002} &= 3/35[4(u_x^2-w_z^2)+2(u_x-w_z)(u_x+v_y+w_z)\\
&\quad +(v_x+u_y)^2-(v_z+w_y)^2]\\
\mu_{011} &= 3/35[3(v_z+w_y)(1/3\,u_x+v_y+w_z)+(u_y+v_x)(u_z+w_x)]\\
\mu_{101} &= 3/35[3(w_x+u_z)(u_x+1/3\,v_y+w_z)+(v_z+w_y)(v_x+u_y)]\\
\mu_{110} &= 3/35[3(u_y+v_x)(u_x+v_y+1/3\,w_z)+(w_x+u_z)(w_y+v_z)].
\end{aligned}\right\} \quad (13)$$

These equations solve our problem for the case of radial velocities, since the corresponding values of the corrections to the elements of the velocity ellipsoid may be easily found. The case of proper motions may also be treated in the same manner.

In practice, however, the inverse problem is more interesting, *i.e.* knowing the observed moments N_{ijk} and the parameters of streaming u_x, u_y, . . . w_z, to determine the true moments N_{ijk}^0 and thus determine the true ellipsoid.

6. In order to try out our method and to see what is in practice the amount of the above corrections we shall apply it to a numerical example. For a strict applicability of the method we require that the velocity ellipsoid and galactic rotation should be determined for the same group of stars. A determination of galactic rotation for different spectral and distance stellar groups from radial velocities has been made by Professor B. Lindblad.* Below are reproduced his results for four groups.

TABLE I

Group	K	l_0	A	$\bar{r}$
		°		
B0–B5	+4·3 ± 0·3	324	0·006	244
B7–B9	+3·6 ± 0·3	334	·013	165
A1	−1·0 ± 2·4	319	·033	163
F–K1	−2·5 ± 0·7	346	·021	190

The first column indicates the spectral interval, the sign 1 for the two last groups indicates that only the stars with proper motion $< 0''{\cdot}040$ were included. This practically eliminates all dwarfs, which, as is well known, do not show any appreciable rotation owing to their nearness. The second column gives the K-term, the third gives the longitude of the galactic centre, the fourth the deduced value of Oort's coefficient, and the fifth the estimated average distance computed from the mean parallax $\bar{\pi}$ according to the formula

$$\bar{r} = \frac{1{\cdot}27}{\bar{\pi}}.$$

Although Lindblad derives also the ellipsoids, our method is not quite applicable to his results, since before proceeding to the ellipsoid determination he has freed the stellar velocities from the average effect of galactic

* *M.N.*, **90**, 5, 1930 (*Stockholm Medd.*, No. 5).

rotation. There remains then only the effect of dispersion in distances, but this amounts only from 1/16 to 1/4 of the full distance-effect according to the assumption made on the density function in our vicinity.*

On the other hand a detailed study of space motions of 4233 stars has been made by R. Wilson and H. Raymond.† They classify these stars into various sub-groups of which the groups Oe–B5, B7–A3, A5–F5 and F8–G5 giants correspond fairly well to the above four Lindblad groups. Assuming their identity we may compute the values of the galactic dynamical coefficients u_x, u_y, v_x, v_y which according to (6) define the corrections for the distance-effect. Here we may confine ourselves to the case of plane-parallel motion, whereby we put

$$u_z = v_z = w_x = w_y = w_z = 0.$$

Formula (6) then reduces to a set of three equations :

$$\left.\begin{aligned} N_{200} &= N_{200}{}^0 + \rho^2/5\ (u_x{}^2 + u_y{}^2), \\ N_{110} &= N_{110}{}^0 + \rho^2/5\ (u_x v_x + u_y v_y), \\ N_{020} &= N_{020}{}^0 + \rho^2/5\ (v_x{}^2 + v_y{}^2). \end{aligned}\right\} \qquad (14)$$

If we write down galactic rotation terms in radial and transverse velocities for stars in the galactic equator in the form

$$v_r = \bar{r}(B' + A_1 \sin 2l + C \cos 2l),$$
$$v_l = \bar{r}(B - C \sin 2l + A_1 \cos 2l)$$

(the subscript 1 being added to A in order to distinguish it from Oort's coefficient A which, obviously, is equal to $(A_1{}^2 + C^2)^{1/2}$), then u_x, u_y, v_x, v_y are defined by the formulæ ‡

$$u_x = B' + C\,; \qquad v_x = A + B,$$
$$u_y = A_1 - B\,; \qquad v_y = B' - C.$$

Except B all other coefficients are provided by the discussion of radial velocities, and their values based upon the data of Table I are given below in Table II. For deriving B we should either apply to proper motions or get it from radial velocities in a somewhat indirect way, using the Lindblad-Oort formula connecting the galactic velocity dispersions σ_1 and σ_2 with Oort's coefficients, namely,

$$\sigma_2{}^2/\sigma_1{}^2 = 1 - \frac{A}{A - B},$$

whence

$$B = -A\sigma_2{}^2/\sigma_1{}^2(1 - \sigma_2{}^2/\sigma_1{}^2)^{-1}.$$

The application of the last formula involves evidently all the difficulties set forth by the assumptions underlying the derivation of this formula.‡ But apart from this, the value of B thus obtained is in some respects not inferior to the value obtained directly, since the latter is influenced to an unknown extent by systematic errors in proper motions. And lastly, since we have

* I wish here to acknowledge my indebtedness to Dr. H. Nordström, now at Stockholm, for his helpful criticism and suggestions relative to this point.

† *A.J.*, 944, 1930.

‡ See O. Heckmann's note in *M.N.*, **95**, 67, 1936.

in view the order of magnitude of the distance-effect rather than its exact value, we may satisfy ourselves with this theoretical value of B. These values for our groups are also reproduced in Table II. The last column of this table gives the value of $\rho = 4/3\,\bar{r}$.

TABLE II

Group	A_1	C	B'	σ_2/σ_1	B	ρ
Bo–B5	+·0019	−·0057	+·0176	·90	−·0256	325
B7–B9	+·0080	−·0102	+·0218	·85	−·0338	220
A1	+·0046	−·0327	−·0061	·66	−·0225	217
F–K1	+·0085	−·0099	−·0132	·78	−·0202	253

From this we get the following values of u_x, u_y, v_x, v_y and then by (14) also of μ_{200}, μ_{110}, μ_{020}.

TABLE III

	u_x	u_y	v_x	v_y	μ_{200}	μ_{110}	μ_{020}
Bo–B5	·0119	·0275	−·0237	·0233	·000180	·000072	·000221
B7–B9	·0116	·0418	−·0258	·0320	·000376	·000208	·000338
A1	−·0388	·0301	−·0209	·0266	·000482	·000322	·000229
K1	−·0231	·0287	−·0117	−·0033	·000271	·000035	·000030

It is not easy to find a determination of velocity ellipsoids which have been made for exactly the same groups of stars. As stated above, the best I could find is afforded by the computations made by Wilson and Raymond which are based on space velocities of 4233 stars.

Now, expressed in galactic co-ordinates, their results for the velocity ellipsoids are in Table IV (l, b denoting galactic co-ordinates * and σ the velocity dispersion, the subscripts 1, 2, 3 referring to the three axes).

TABLE IV

Wilson and Raymond Velocity Ellipsoids

	l_1	b_1	σ_1	l_2	b_2	σ_2	l_3	b_3	σ_3
	°	°		°	°		°	°	
Oe–B5	348·0	+15·8	13·5	81·7	+13·2	11·4	209·6	+69·2	9·2
B7–A3	349·9	− 4·0	16·5	80·2	− 5·3	11·4	42·5	+83·4	8·1
A5–F3	345·7	− 0·4	26·2	75·7	− 4·1	18·1	69·0	+85·9	12·5
F8–G5gi	353·4	+ 2·6	26·5	84·0	+12·7	20·6	252·1	+76·9	14·3

The first two axes are very nearly in the galactic plane. Thus we shall disregard in the following computation their inclinations and consider only the projection of the ellipsoids upon the galactic plane. Denoting by l_0 the longitude of the vertex, we then can compute the values of the observed moments N_{200}, N_{110} and N_{020} according to the formulæ :

$$N_{200} = \frac{\sigma_1^2 + \sigma_2^2}{2} + \frac{\sigma_1^2 - \sigma_2^2}{2} \cos 2l_0\,; \qquad N_{110} = \frac{\sigma_1^2 - \sigma_2^2}{2} \sin 2l_0\,;$$

$$N_{020} = \frac{\sigma_1^2 + \sigma_2^2}{2} - \frac{\sigma_1^2 - \sigma_2^2}{2} \cos 2l_0.$$

* Referred to the galactic pole $\alpha = 190°$, $\delta = +28°$.

Multiplying then the values of μ_{200}, μ_{110} and μ_{020} given in Table III by the corresponding values of ρ^2 given in the last column of Table II we obtain the corrections which we have to subtract from the observed moments in order to obtain the true moments N_{200}^0, N_{110}^0, N_{020}^0. These in turn permit us to compute the corrected longitude of the vertex l_0' and the true dispersions $\sigma_1'^2$ and $\sigma_2'^2$ according to the formulæ :

$$\tan 2l_0' = \frac{2N_{110}^0}{N_{200}^0 - N_{020}^0}; \qquad \sigma_1'^2 = \frac{N_{200}^0 + N_{020}^0}{2} + \frac{N_{200}^0 - N_{020}^0}{2} \cos 2l_0';$$

$$\sigma_2'^2 = \frac{N_{200}^0 + N_{020}^0}{2} - \frac{N_{200}^0 - N_{020}^0}{2} \cos 2l_0'.$$

The results are given below in Table V.

TABLE V

	l_0	N_{200}	N_{110}	N_{020}	N_{200}^0	N_{110}^0	N_{020}^0	l_0'	σ_1'	σ_2'
	°							°		
Oe–B5	348	179·9	−10·6	132·3	160·8	−18·2	108·9	342	12·9	10·2
B7–A3	350	267·9	−24·3	134·3	249·9	−34·3	118·1	346	16·1	10·5
A5–F3	346	665·4	−84·2	348·6	642·7	−99·4	337·8	343	25·9	17·6
F8–G5gi	353	698·1	−33·6	428·5	680·8	−35·8	426·6	352	26·2	20·5

We see on comparing the corrected and observed values of the longitude of the vertex that it has decreased by an angle ranging from 1° to 6°, whereas the maximum decrease of dispersions does not quite reach 1 km. per sec.

The smallness of these corrections is somewhat unexpected, since, theoretically speaking, the corrected values of Wilson and Raymond should be very close to the values of Lindblad since the latter were computed from stellar velocities corrected for the galactic rotation effect at the start.

TABLE VI

Lindblad Velocity Ellipsoids

	l_0	σ_1	σ_2	N_{200}	N_{110}	N_{020}
	°					
Bo–B5	289	10·9	9·8	98·2	− 6·8	20·1
B8–B9	275	12·4	10·5	111·1	− 4·2	130·2
A1	21	18·6	12·4	322·9	+64·5	15·0
F–K1	335	17·5	13·6	284·6	−46·3	272·5

We see that Lindblad's σ_1 and σ_2 are in the average 1·4 times smaller than the corresponding values obtained by Wilson and Raymond. For the second-order moments N_{200}, N_{110}, N_{020}, which define the dispersions, this ratio amounts even to 2·2. This large discrepancy is very probably not real and is to a large extent explained by the particular method of computing the moments adopted by Lindblad. Indeed, in order to eliminate as far as possible the influence of high velocity stars which usually disturb very much the values of the dispersions, he followed a rather indirect method of their derivation, computing instead the average absolute residual velocity in each of the 48 standard areas on the celestial sphere and multiplying

afterwards these numbers by a theoretical factor equal to 1·253.* This procedure would be quite correct only if the velocity distribution were Gaussian. But it is just the presence of high velocities which causes the so-called positive excess in the observed velocity distribution. The larger the dispersion, the larger is the excess and therefore also the error committed by assuming the distribution to be Gaussian. An effect of this kind is actually apparent, since the difference between Lindblad on the one hand, and Wilson and Raymond on the other, increases quite regularly with the value of the dispersion. This may be seen from the following table where the ratio of the dispersions σ_1 (in the sense Wilson-Raymond : Lindblad) is given:—

	B0–B5	B7–B9	A1	F–K1
$\frac{\sigma_1(\text{W.-R.})}{\sigma_1(\text{L.})}$	1·24	1·33	1·41	1·51
$\sigma_1(\text{W.-R.})$	13·5	16·5	26·2	26·5

It is probable therefore that the quite unusual values of the longitude of the vertex obtained by Lindblad, which deviate by as much as 70° from the standard direction, may be explained by his method of determining N_{200}, N_{110}, N_{020}.

In any case we see that the ellipsoids are very sensitive to the particular method used in their derivation, and thus it appears very desirable that investigators in order to obtain comparable results should agree in the choice of the method of computation of the ellipsoids as well also as in the manner of dissecting stars into spectral groups. In view of the disturbing action of the high velocity stars, methods other than that of Charlier which do not involve the computation of second-order moments are perhaps preferable.† It may be shown also that the distance-effect discussed above is then practically negligible. But on the other hand we may observe that what we really are determining by computing a velocity ellipsoid is just the dispersions themselves, and thus any artificial process of their computation which avoids the computation of average square velocities is likely not to give what we really are looking for. At the same time it should be remembered that just the mean square velocities (or rather their derivatives with respect to co-ordinates) enter into the hydrodynamical equations of motion, and thus, irrespective of the approximation to the actual velocity distribution which the ellipsoid provides, they always remain an important dynamical factor.

Astronomical Observatory,
Pulkovo :
1936 *May.*

* *Loc. cit.*, p. 507.

† See, *e.g.*, Strömberg's method, *Ap. J.*, **56**, 265, 1922, or Mme. et M. H. Mineur, *Bull. Astronomique, Mémoires et variétés*, t. vii, f. viii, 1931.

THE FOURIER CONSTANTS OF A DRIFT-CURVE.

A. Fletcher, M.A., Ph.D.

1. In the analysis of the proper motions of any region of the sky on the two-drift hypothesis, the fundamental property of a single drift containing N stars is that the number of proper motions belonging to the drift and inclined to the drift-axis at angles between θ and $\theta+d\theta$ is given by $NFd\theta$, where

$$F=\frac{h^2}{\pi}\int_0^\infty e^{-h^2(V^2+T^2-2VT\cos\theta)}TdT, \tag{1}$$

hV being the projected drift-velocity. It has been shown by Eddington * that

$$F=\frac{1}{\pi}e^{-h^2V^2}\left\{\tfrac{1}{2}+\tau e^{\tau^2}\int_0^\tau e^{-x^2}dx+\frac{\pi^{\frac{1}{2}}\tau e^{\tau^2}}{2}\right\}, \tag{2}$$

where $\tau=hV\cos\theta$.

The function F is even with respect to θ, and may be expanded in a Fourier series of the form

$$\pi F=\tfrac{1}{2}A_0+A_1\cos\theta+A_2\cos 2\theta+A_3\cos 3\theta+\ldots,$$

where the coefficients A_m, which are functions of hV, are given by

$$A_m=\int_0^{2\pi}F\cos m\theta d\theta=2\int_0^{\pi}F\cos m\theta d\theta. \tag{3}$$

It follows from the definition of F that

$$A_0=\int_0^{2\pi}Fd\theta=1. \tag{4}$$

It has also been shown by Eddington * that

$$A_1=\tfrac{1}{2}\pi^{\frac{1}{2}}hVe^{-\frac{1}{2}h^2V^2}\{I_0(\tfrac{1}{2}h^2V^2)+I_1(\tfrac{1}{2}h^2V^2)\}, \tag{5}$$

$$A_2=1-(1-e^{-h^2V^2})/h^2V^2, \tag{6}$$

$$A_3=\tfrac{1}{2}\pi^{\frac{1}{2}}hVe^{-\frac{1}{2}h^2V^2}\{I_0(\tfrac{1}{2}h^2V^2)+(1-4/h^2V^2)I_1(\tfrac{1}{2}h^2V^2)\}, \tag{7}$$

where the modified Bessel function $I_\nu(x)$ is defined in the usual way as $i^{-\nu}J_\nu(ix)$.

Let us write $h^2V^2=2b$, and use the formulæ †

$$I_{-\frac{1}{2}}(b)=(2/\pi b)^{\frac{1}{2}}\cosh b, \tag{8}$$

$$I_{\frac{1}{2}}(b)=(2/\pi b)^{\frac{1}{2}}\sinh b, \tag{9}$$

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* *M.N.*, 68, 588, 1908.

† See Watson, *Theory of Bessel Functions*, pp. 53–55, 1922, for the general expressions of $J_\nu(z)$, where ν is half of an odd integer, positive or negative.

$$I_{\frac{3}{2}}(b) = (2/\pi b)^{\frac{1}{2}}(\cosh b - \sinh b/b), \tag{10}$$

$$I_2(b) = I_0(b) - 2I_1(b)/b. \tag{11}$$

We then find that the results (4) to (7) may be expressed in the form

$$A_0 = (\tfrac{1}{2}\pi b)^{\frac{1}{2}}e^{-b}(I_{-\frac{1}{2}} + I_{\frac{1}{2}}) = 1,$$
$$A_1 = (\tfrac{1}{2}\pi b)^{\frac{1}{2}}e^{-b}(I_0 + I_1),$$
$$A_2 = (\tfrac{1}{2}\pi b)^{\frac{1}{2}}e^{-b}(I_{\frac{1}{2}} + I_{\frac{3}{2}}),$$
$$A_3 = (\tfrac{1}{2}\pi b)^{\frac{1}{2}}e^{-b}(I_1 + I_2),$$

the argument of the Bessel functions being b. This suggests a general theorem,

$$A_m = (\tfrac{1}{2}\pi b)^{\frac{1}{2}}e^{-b}\{I_{\frac{1}{2}(m-1)}(b) + I_{\frac{1}{2}(m+1)}(b)\}, \tag{12}$$

which we proceed to prove.

2. In the bracket on the right side of (2) the first two terms are even functions of $\cos\theta$ and the third term is an odd function of $\cos\theta$. Thus the third term corresponds to the coefficients A_1, A_3, . . . and the other terms to A_0, A_2, . . . A proof of (12) by means of (2) for all values of m at once is, consequently, hardly to be expected. Separate proofs for odd and even m are possible, and that for odd m is simple ; but we shall confine ourselves to a proof for all m by means of (1).

3. From (1) and (3) we have

$$A_m = \frac{2h^2}{\pi}e^{-h^2V^2}\int_0^\pi \cos m\theta\left(\int_0^\infty Te^{-h^2T^2}e^{2h^2VT\cos\theta}dT\right)d\theta$$
$$= \frac{2h^2}{\pi}e^{-h^2V^2}\int_0^\infty Te^{-h^2T^2}\left(\int_0^\pi e^{2h^2VT\cos\theta}\cos m\theta d\theta\right)dT.$$

Recalling that for integral m

$$\int_0^\pi e^{x\cos\theta}\cos m\theta d\theta = \pi I_m(x),$$

and writing $hT = t$, we have

$$A_m = 2e^{-h^2V^2}\int_0^\infty te^{-t^2}I_m(2hVt)dt.$$

Since $h^2V^2 = 2b$, comparison with (12) now shows that we require to prove the relation

$$\int_0^\infty te^{-t^2}I_m(\sqrt{8bt})dt = (\pi b/8)^{\frac{1}{2}}e^b\{I_{\frac{1}{2}(m-1)}(b) + I_{\frac{1}{2}(m+1)}(b)\}, \tag{13}$$

where m is a positive integer or zero. The relation is in fact true when $m > -2$ (which secures the convergence of the integral at its lower limit), and also in the trivial case when m is a negative odd integer.

4. Consider first the left side of (13). Writing $t^2 = x$, we have, if $m > -2$,

$$\int_0^\infty te^{-t^2}I_m(\sqrt{8b}t)dt = \tfrac{1}{2}\int_0^\infty e^{-x}I_m(\sqrt{8bx})dx$$

$$= \tfrac{1}{2}\int_0^\infty e^{-x}\sum_{r=0}^\infty \frac{(2bx)^{r+\frac{1}{2}m}}{r!\,\Gamma(r+m+1)}dx$$

$$= \tfrac{1}{2}\sum_{r=0}^\infty \frac{(2b)^{r+\frac{1}{2}m}}{r!\,\Gamma(r+m+1)}\int_0^\infty e^{-x}x^{r+\frac{1}{2}m}dx$$

$$= \tfrac{1}{2}\sum_{r=0}^\infty \frac{(2b)^{r+\frac{1}{2}m}\Gamma(r+\frac{1}{2}m+1)}{r!\,\Gamma(r+m+1)}. \tag{14}$$

The infinite integral on the left side of (13) is of a type which has been considered by Weber, Hankel and later writers, and the series (14) is a particular case of one given by Hankel.* It has now to be identified with the expression on the right side of (13).

5. We require a form of Kummer's relation, namely,†

$$e^b I_\nu(b) = \frac{1}{\sqrt{\pi}}\sum_{r=0}^\infty \frac{(2b)^{r+\nu}}{r!}\frac{\Gamma(r+\nu+\frac{1}{2})}{\Gamma(r+2\nu+1)}. \tag{15}$$

Thus

$$(2\pi b)^{\frac{1}{2}}e^b I_{\frac{1}{2}(m-1)}(b) = \sum_{r=0}^\infty \frac{(2b)^{r+\frac{1}{2}m}}{r!}\frac{\Gamma(r+\frac{1}{2}m)}{\Gamma(r+m)}. \tag{16}$$

Similarly

$$(2\pi b)^{\frac{1}{2}}e^b I_{\frac{1}{2}(m+1)}(b) = \sum_{s=0}^\infty \frac{(2b)^{s+\frac{1}{2}m+1}}{s!}\frac{\Gamma(s+\frac{1}{2}m+1)}{\Gamma(s+m+2)}$$

$$= \sum_{r=0}^\infty \frac{r(2b)^{r+\frac{1}{2}m}}{r!}\frac{\Gamma(r+\frac{1}{2}m)}{\Gamma(r+m+1)}, \tag{17}$$

where we have written $r = s+1$, but have taken the summation in (17) to begin at $r = 0$, since the term having $r = 0$ vanishes. We thus have, adding (16) and (17),

$$(2\pi b)^{\frac{1}{2}}e^b\{I_{\frac{1}{2}(m-1)}(b) + I_{\frac{1}{2}(m+1)}(b)\}$$

$$= \sum_{r=0}^\infty \frac{(2b)^{r+\frac{1}{2}m}\Gamma(r+\frac{1}{2}m)}{r!\,\Gamma(r+m+1)}\{(r+m)+r\}$$

$$= 2\sum_{r=0}^\infty \frac{(2b)^{r+\frac{1}{2}m}\Gamma(r+\frac{1}{2}m+1)}{r!\,\Gamma(r+m+1)}. \tag{18}$$

From (14) and (18) the relation (13) follows immediately, and consequently the general result (12) is established.

It should perhaps be pointed out that the form in which Kummer's relation has been written in (15) implies slight modifications when ν is half a negative odd integer. If $m > -2$, the only relevant case is $\nu = -\frac{1}{2}$, arising from $m = 0$ in (16). The modification then is that in the first term, for which $r = 0$, $\Gamma(\frac{1}{2}m)/\Gamma(m)$ is to be replaced by its limit as $m \to 0$, which is 2.

* See Watson, *Theory of Bessel Functions*, p. 394, 1922.

† *Cf.* Watson, *ibid.*, p. 191, 1922.

It is easily verified that the result (18) survives, but the point is not very important, since the case $m=0$ is one for which the truth of (12) has already been noticed in section 1.

6. We now derive the series for A_m in powers of hV, since this is of use when hV is not too large. We have the relation

$$e^{-b}I_\nu(b)=\frac{1}{\sqrt{\pi}}\sum_{r=0}^{\infty}(-)^r\frac{(2b)^{r+\nu}}{r\,!}\,\frac{\Gamma(r+\nu+\tfrac{1}{2})}{\Gamma(r+2\nu+1)},$$

which differs from (15) only in the signs. A process similar to that of section 5 gives

$$(2\pi b)^{\frac{1}{2}}e^{-b}\{I_{\frac{1}{2}(m-1)}(b)+I_{\frac{1}{2}(m+1)}(b)\}$$
$$=\sum_{r=0}^{\infty}(-)^r\frac{(2b)^{r+\frac{1}{2}m}\Gamma(r+\frac{1}{2}m)}{r\,!\,\Gamma(r+m+1)}\{(r+m)-r\}.$$

We thus have, from (12),

$$A_m=\frac{m}{2}\sum_{r=0}^{\infty}(-)^r\frac{(hV)^{2r+m}\Gamma(r+\frac{1}{2}m)}{r\,!\,\Gamma(r+m+1)}.$$

This series is particularly convenient when m is even. In particular,

$$A_4=\frac{2}{4!}h^4V^4-\frac{4}{5!}h^6V^6+\frac{6}{6!}h^8V^8-\ \ldots$$

This series may also be obtained from the equation

$$A_4=1-\frac{4}{h^2V^2}+\frac{6}{h^4V^4}-\left(\frac{2}{h^2V^2}+\frac{6}{h^4V^4}\right)e^{-h^2V^2},$$

which follows from (12), (10) and the formula

$$I_{\frac{5}{2}}(b)=(2/\pi b)^{\frac{1}{2}}\{(1+3b^{-2})\sinh b-3b^{-1}\cosh b\}.$$

7. It follows from (4) and (12) that A_0, A_1, A_2, A_3, . . . are proportional to

$$I_{-\frac{1}{2}}+I_{\frac{1}{2}},\qquad I_0+I_1,\qquad I_{\frac{1}{2}}+I_{\frac{3}{2}},\qquad I_1+I_2,\ \ldots,$$

and A_0 is unity. This would provide a very simple means of computing the coefficients A_m if tables of all the Bessel functions required were available. For example, when $hV=2$, $b=2$, we have

$$I_{-\frac{1}{2}}=2{\cdot}12259,\qquad I_0=2{\cdot}27959,$$
$$I_{\frac{1}{2}}=2{\cdot}04624,\qquad I_1=1{\cdot}59064,$$
$$I_{\frac{3}{2}}=1{\cdot}09947,\qquad I_2=0{\cdot}68895.$$

By simple additions we form the sequence

$$4{\cdot}16883,\qquad 3{\cdot}87023,\qquad 3{\cdot}14571,\qquad 2{\cdot}27959,$$

and dividing throughout by the first member we obtain, besides $A_0=1$,

$$A_1=0{\cdot}9284,\qquad A_2=0{\cdot}7546,\qquad A_3=0{\cdot}5468,$$

in agreement with the values tabulated by Eddington.* Similarly, we find that $A_4 = 0{\cdot}3590$.

A similar process could be used if tables of $e^{-b}I_\nu(b)$ were available for all relevant values of ν. Very unfortunately, neither $I_\nu(b)$ nor $e^{-b}I_\nu(b)$ appears to have been tabulated when 2ν is an odd integer, and the numerical values given in the first column above have had to be specially computed.

Department of Applied Mathematics,
University of Liverpool:
1936 *September* 9.

OBSERVATIONS OF COMET PELTIER (1936 *a*).

W. H. Steavenson.

The observations of position which follow (Table I) were made with the 20½-inch Hindle reflector. A cross-bar micrometer was used, in conjunction with a chronograph.

The positions of the comparison stars, given in Table II, include corrections for proper motion where the necessary data were available. Only one star was used on each occasion. The observations of the comet have not been corrected for parallax or differential refraction. The position of the observatory is:

W. Long. $0^h\ 00^m\ 23^s{\cdot}10$.
N. Lat. 51° 26′ 09″·9.

I am indebted to Dr. A. C. D. Crommelin and Mr. P. J. Melotte for help in deriving the positions of some of the comparison stars.

TABLE I

	U.T.	α (1936·0)	δ (1936·0)	Comet *minus* Star Δα	Comet *minus* Star Δδ	Number of Observations
	d h m s	h m s	° ′ ″	m s	′ ″	
1	1936 May 18 21 55·6	23 55 49·47	+73 10 44·2	−0 20·64	+ 0 18·6	5
2	19 21 57·8	23 57 7·41	+73 4 50·6	+0 57·30	− 5 35·0	4
3	21 21 52·6	23 59 29·80	+72 53 27·7	−0 35·82	+ 4 22·8	10
4	28 22 4·7	0 5 51·95	+72 13 47·7	+2 6·25	−11 19·5	5
5	June 1 23 3·7	0 8 7·48	+71 50 50·6	+0 2·90	+ 8 40·5	5
6	2 21 53·4	0 8 31·69	+71 45 14·2	+0 27·11	+ 3 4·1	6
7	9 22 6·9	0 9 54·81	+71 2 10·8	+0 8·97	− 7 24·8	5
8	15 23 1·2	0 7 48·92	+70 19 41·8	−1 8·72	− 0 56·1	2
9	July 5 22 26·9	23 48 36·86	+66 14 33·2	+0 9·28	+ 0 35·9	10
10	15 22 51·6	23 24 46·05	+61 4 10·8	−0 30·41	− 2 53·4	5
11	25 22 46 45	22 44 2·98	+46 22 46·8	−0 44·97	− 1 53·2	12
12	26 21 46 54	22 38 55·93	+43 38 24·2	+0 2·85	− 1 34·6	12
13	30 1 0 55	22 20 23·51	+31 17 19·1	−0 1·43	+ 6 41·2	10
14	August 1 23 5 45	22 0 43·53	+13 28 8·9	−0 35·59	− 2 44·1	10
15	3 23 35 14	21 45 45·13	− 2 15 34·8	−0 6·59	+ 4 43·1	10
16	4 23 39 29	21 37 57·34	−10 26 30·1	+0 20·01	+ 2 51·7	10

* *M.N.*, **68**, 592, 1908.

TABLE II

	B.D. No.	Mag.	Adopted α (1936·0)	Adopted δ (1936·0)	Authority
	°		h m s	° ′ ″	
1	+72 1132	9·3	23 56 10·11	+73 10 25·6	Greenwich Astr. Cat.
2	+72 1132	9·3	23 56 10·11	+73 10 25·6	,, ,,
3	+72 1136	7·0	0 0 5·62	+72 49 4·9	,, ,,
4	+71 1250	7·8	0 3 45·70	+72 25 7·2	,, ,,
5	+71 5	9·5	0 8 4·58	+71 42 10·1	,, ,,
6	+71 5	9·5	0 8 4·58	+71 42 10·1	,, ,,
7	+70 3	9·5	0 9 45·84	+71 9 35·6	,, ,,
8	+69 8	9·2	0 8 57·64	+70 20 37·9	,, ,,
9	+65 1954	9·1	23 48 27·58	+66 13 57·3	,, ,,
10	+60 2550	7·9	23 25 16·46	+61 7 4·2	Helsingfors Astr. Cat.
11	+45 4054	8·2	22 44 47·95	+46 24 40·0	Bonn A.G. Cat.
12	+43 4270	9·0	22 38 53·08	+43 39 58·8	,, ,,
13	+30 4698	8·0	22 20 24·94	+31 10 37·9	Greenwich 1910. Berlin 1920.
14	+13 4840	8·6	22 1 19·12	+13 30 53·0	Bordeaux 1907. Skjellerup 1875. A.G. 1875.
15	− 2 5642	9·0	21 45 51·72	− 2 20 17·9	Abbadia Cat.
16	−10 5735	9·1	21 37 37·33	−10 29 21·8	Cambridge A.G. Cat.

PRELIMINARY VALUES OF THE VARIATION OF LATITUDE AT GREENWICH, 1935·0–1936·4

(Communicated by the Astronomer Royal)

This paper contains the preliminary values of the latitude variation derived from observations made with the Cookson floating telescope from the beginning of 1935 to the conclusion of the observing programme commenced in 1911.

The star places used were those derived from the final discussion of the observations for the period 1911–1936.

The Cookson telescope was dismantled on 1936 June 8. It has been thoroughly overhauled and has been erected in the Christie Enclosure, close to the Reversible Transit Circle. It will be employed on a modified programme, planned solely for the determination of the variation of latitude.

The following table gives the values of the variation (mean latitude *minus* observed latitude) :—

Date	Variation	Number of Pairs
	″	
1935·005	−·010	77
·187	+·114	77
·231	+·086	87
·278	+·133	98
·340	+·147	84
·429	+·045	77
·492	+·098	86

Date	Variation	Number of Pairs
	″	
1935·539	+·018	92
·573	−·038	93
·619	−·018	92
·676	−·072	85
·724	−·061	94
·773	−·090	89
·819	−·040	90
·884	−·013	74
1936·023	+·097	90
·099	+·070	77
·158	+·136	80
·244	+·101	92
·310	+·145	100
·363	+·040	91
1936·397	+·178	79

From a smooth curve drawn to represent the observed results as closely as possible, the following corrections, applicable to observed north polar distances, were derived :—

	″		″
1935·0	+·01	1935·8	−·06
·1	+·07	·9	00
·2	+·11	1936·0	+·06
·3	+·12	·1	+·10
·4	+·10	·2	+·12
·5	+·04	·3	+·13
·6	−·03	1936·4	+·12
·7	−·07		

Royal Observatory, Greenwich :
1936 *July*.

THE PROFILE OF THE ABSORPTION LINES IN ROTATING STARS, TAKING INTO ACCOUNT THE VARIATION OF IONIZATION DUE TO CENTRIFUGAL FORCE.

P. Swings and S. Chandrasekhar.

1. The investigations by Elvey * and Miss Westgate † have shown that a very great proportion of the stars of classes B and A have rotational velocities of the order of 100 km./sec., and that 5 per cent. of the early type stars have velocities larger than 200 km./sec. In several papers ‡ Struve has indicated that in certain cases the centrifugal acceleration V^2/R may be of the same order as the surface gravity g, the ratio $\epsilon = V^2/Rg$ having in quite reasonable stars a value approaching 0·5. Actually, a calculation of the ratio ϵ for many B and A type stars of large rotational velocities gives values of the order of 0·4. The importance of these facts, in view of the mechanical instability of certain rotating stars as argued by Struve, is clear.

* C. T. Elvey, *Ap. J.*, **71**, 221, 1930. † C. Westgate, *Ap. J.*, **78**, 46, 1933.
‡ O. Struve, *Ap. J.*, **72**, 1, 1930 ; *Pop. Astr.*, **43**, 492, 1935.

In a recent paper * Hynek has dealt with the effect of rotation upon the state of ionization ; his work shows that, statistically, the ionization is not materially different in rotating stars. But it has seemed interesting to us to investigate theoretically the special case of the rotating stars with high values of ϵ ; actually, all the numerical examples which will be considered later will correspond to $\epsilon = 0{\cdot}4$, which is a quite reasonable value for certain stars having high rotational velocities.

The gravity at a given latitude on the star is reduced by a definite amount by the centrifugal acceleration, the effect being a maximum at the equator and absent at the poles. Consequently, the ionization must vary as a function of the stellar latitude.

For high values of ϵ this effect may give rise to differences between the profiles of lines of neutral and ionized atoms. It is the object of this paper to discuss briefly this question from the theoretical side.

An investigation of this kind may be useful either for the interpretation of irregularities which should possibly be observable in accurate profiles when ϵ is known, or for the detection of stars having a high value of ϵ.

We assume the angular velocity to be constant and the star to be spherical. On the other hand, we suppose that the intrinsic width of the line is very small compared with that due to Doppler broadening ; this is obviously justifiable if we consider high rotational velocities. We shall only compare the profiles of lines of neutral and ionized atoms.

2. *Standard Cases.*†—The expression " standard profiles " will mean the profiles obtained without considering the latitude effect of rotation on the ionization.

Let V be the velocity at the equator ;
ν_0 the wave number of the centre of the line ;
ν the wave number at any point of the profile ;
v the velocity corresponding to the wave number ν ;
θ the colatitude on the star.

A. *Standard profile S_{ND}, neglecting the darkening towards the limb.* It is easily seen ‡ that *the contour is an integration over flat-topped elementary*

* J. A. Hynek, *Ap. J.*, 83, 476, 1936.

† For a general summary on this subject, see Rosseland, *Theoretical Astrophysics*, pp. 202–215, 1936. We have, however, used a different method of integration, which was more suitable for our particular problem.

‡ The contribution of the region between θ and $\theta + d\theta$ is proportional to

$$R \sin \theta d\phi \, . \sin \phi \, . \, R \sin \theta d\theta, \qquad (a)$$

the angle ϕ being the azimuthal angle connected to ν and θ by

$$\nu = \nu_0 \left(1 + \frac{V}{c} \sin \theta \cos \phi \right). \qquad (b)$$

Differentiation of (*b*) gives

$$d\nu = \nu_0 \frac{V}{c} \sin \theta \sin \phi d\phi ;$$

consequently (*a*) is

$$I_0 \, . \, d\nu \, . \sin \theta d\theta.$$

contours of intensity $I_0 \cdot d\nu \sin\theta \cdot d\theta$, *arising between* θ *and* $\theta + d\theta$, *and extending from* θ_1 *to* $\pi/2$, θ_1 being given by $\frac{V}{c}\sin\theta_1 = \frac{\nu - \nu_0}{\nu_0}$.

We have thus

$$S_{ND}d\nu = I_0 \cdot d\nu\left[1 - \frac{c^2}{V^2}\left(\frac{\nu-\nu_0}{\nu_0}\right)^2\right]^{\frac{1}{2}}. \tag{1}$$

We shall use the following abbreviation :—

$$G(\nu) = \left[1 - \frac{c^2}{V^2}\left(\frac{\nu-\nu_0}{\nu_0}\right)^2\right]^{\frac{1}{2}} = \left(1 - \frac{v^2}{V^2}\right)^{\frac{1}{2}}. \tag{2}$$

Omitting the constant factor, we write this

$$S_{ND} = G(\nu). \tag{1'}$$

B. *Standard profile* S_D, *taking into account the darkening towards the limb.* If we assume the star to be darkened towards the limb, u being the coefficient of darkening, we have now *

$$S_D = \int_{\theta_1}^{\pi/2} \sin\theta\left\{1 - u + u\sin\theta\left[1 - \left(\frac{\nu-\nu_0}{\nu_0}\right)^2 \cdot \frac{c^2}{V^2\sin^2\theta}\right]^{\frac{1}{2}}\right\} d\theta.$$

The result is

$$S_D = G(\nu)\left[(1-u) + \frac{\pi}{4}uG(\nu)\right]. \tag{3}$$

3. *Actual Problem.*—Inside the integrals we have to introduce in each elementary contour a weighting factor, $w(\theta)$, which corresponds to the number of atoms contributing to the absorption line, this number being now a function of the angle θ.

Let us consider a single-stage ionization and let N_1 and N_0 be the proportions of ionized and neutral atoms. We use the equilibrium equation

$$\frac{N_1}{N_0} = \frac{K}{P}, \tag{4}$$

and assume the equilibrium constant K to be independent of θ.

At the angle θ the surface-gravity g is reduced to

$$g_{eff} = g\left(1 - \frac{V^2}{Rg}\cdot\sin^2\theta\right) = g(1 - \epsilon\sin^2\theta). \tag{5}$$

We shall consider two cases :

(*a*). P proportional to g_{eff}, thus giving †

$$P = P_0(1 - \epsilon\sin^2\theta); \tag{6}$$

(*b*) P proportional to $\sqrt{g_{eff}}$,

$$P = P_0\sqrt{1 - \epsilon\sin^2\theta}. \tag{7}$$

* We neglect the constant factor I_0.

† In this investigation, θ terms independent of ϵ are ignored and a mean value assumed for τ.

If we assume a constant optical depth τ, these two assumptions correspond respectively to the values 0 and 1 of the coefficient n introduced by Unsöld in his investigation of stellar atmospheres ; * the first case is probably the most important one.

Equation (4) gives

$$\text{Case } a \begin{cases} N_1 = \dfrac{K}{K + P_0(1 - \epsilon \sin^2 \theta)}, \\ N_0 = \dfrac{P_0(1 - \epsilon \sin^2 \theta)}{K + P_0(1 - \epsilon \sin^2 \theta)}. \end{cases} \tag{6'}$$

$$\text{Case } b \begin{cases} N_1 = \dfrac{K}{K + P_0 . \sqrt{1 - \epsilon \sin^2 \theta}}, \\ N_0 = \dfrac{P_0 \sqrt{1 - \epsilon \sin^2 \theta}}{K + P_0 \sqrt{1 - \epsilon \sin^2 \theta}}. \end{cases} \tag{7'}$$

We also have to assume the relation between N and w ; owing to the fact that we consider here Doppler effects, we shall take w proportional † to N.

4. *First Problem.—P proportional to g_{eff}; no consideration of darkening towards the limb ; K very large.*

N_1 is then nearly unity, and the contour of a line of the ionized element is evidently the standard contour $S_{ND} = G(v)$. For the lines of the neutral atom the contour is

$$I dv = I_0 . dv . \frac{P_0}{K} \int_{\theta_1}^{\frac{\pi}{2}} \sin \theta . (1 - \epsilon \sin^2 \theta) d\theta.$$

Neglecting constant factors, this gives ‡

$$(\text{Contour})_0' = G(v)\left[1 - \epsilon + \frac{\epsilon}{3} G^2(v)\right]. \tag{8}$$

Numerical Example.—In Table I we give the values of I as a function of v/V, in the particular case $\epsilon = 0{\cdot}4$ for the standard contour S_{ND} and for $(\text{contour})_0'$, both of them being reduced to $I = 1$ at $v/V = 0$.

Supposing that this first problem has a real physical meaning,§ an examination of Table I leads to the following conclusions :—

(1) When comparing lines of neutral and ionized atoms the widths must really differ little.

(2) Where the contour still corresponds to 44 per cent. of the central

* Unsöld, *Zs. f. Astroph.*, 1, 138, 1930 ; equation $\tau = \dfrac{k_0}{g} . \dfrac{P^{n+1}}{n+1}$.

† The introduction of the square-root law $w \propto \sqrt{N}$ would lead to complicated integrals, but no theoretical difficulty. For numerical applications it would be possible to reduce it to the problems considered here.

‡ The notations $(\text{contour})_0$ and $(\text{contour})_1$ respectively refer to the neutral and to the ionized atoms.

§ It will be seen later on that all the other problems give similar results.

TABLE I

$\frac{v}{V}$	S_{ND}	$(\text{Contour})_0'$	$\frac{(\text{Contour})_0'}{S_{ND}}$
0·0	1	1	1
·2	·98	·973	·99
·4	·916	·89	·971
·6	·8	·748	·935
·8	·6	·530	·883
·9	·436	·372	·853

intensity, $(\text{contour})_0'$ is reduced by 15 per cent. compared with the standard contour (or with the contour of the ionized lines); this seems to be within the limits of photometric observations.

(3) In any case, sufficiently high dispersion and great photometric care will be required in order to show the effect conspicuously.

(4) The figures in Table I correspond to $\epsilon = 0{\cdot}4$, which is rather an extreme value, *i.e.* the effect becomes observable only for very fast rotating stars.

5. *Second Problem.—P proportional to g_{eff}; with consideration of darkening towards the limb; K very large.*

For the ionized lines the contour is of course the standard contour S_D given by formula (3). For the neutral lines the integral has to be taken over elementary contours of intensity

$$\sin\theta\left\{1-u+u\sin\theta\left[1-\left(\frac{\nu-\nu_0}{\nu_0}\right)^2\cdot\frac{c^2}{V^2\sin^2\theta}\right]^{\frac{1}{2}}\right\}(1-\epsilon\sin^2\theta)d\theta.$$

The expression obtained is

$$(\text{Contour})_0'' = (1-u)G(\nu)\left[1-\epsilon+\frac{\epsilon}{3}G^2(\nu)\right]+u\frac{\pi}{4}G^2(\nu)\left[1-\epsilon+\frac{\epsilon}{4}G^2(\nu)\right]. \quad (9)$$

Numerical Example.—$\epsilon = 0{\cdot}4$; $u = 0{\cdot}6$. Table II is similar to Table I.

TABLE II

$\frac{v}{V}$	S_D	$(\text{Contour})_0''$	$\frac{(\text{Contour})_0''}{S_D}$
0·0	1	1	1
·2	·970	·963	·99
·4	·874	·854	·98
·6	·714	·672	·94
·8	·471	·422	·90
·9	·303	·263	·86

The effect of the darkening on the intensities is quite appreciable, but the ratios indicated in the last column are similar to those of Table I, leaving the conclusions unaltered.

6. *Third Problem.—P proportional to g_{eff} ; no consideration of darkening ; N_1 and N_0 of the same order.*

The complete formulæ (6′) have to be introduced as factors w in the integrals ; as we have $N_0 = 1 - N_1$ or $w_0 = 1 - w_1$, the contour for the lines of neutral atoms is immediately obtained when we know the expression for the lines of ions. This is

$$(\text{Contour})_1 = \int_{\theta_1}^{\frac{\pi}{2}} \frac{\sin\theta \, . \, d\theta}{K + P_0(1 - \epsilon \sin^2\theta)}$$

$$= \frac{1}{\sqrt{[K + P_0(1-\epsilon)]P_0\epsilon}} \cdot \tan^{-1} G(\nu) \cdot \left[\frac{P_0\epsilon}{K + P_0(1-\epsilon)}\right]^{\frac{1}{2}}. \quad (10)$$

Numerical Example.—Let us suppose $\epsilon = 0{\cdot}4$ and $K/P_0 = 1$. Table III gives information similar to that of Tables I and II ; $(\text{contour})_1$ and $(\text{contour})_0$ correspond respectively to the ionized and neutral atoms.

TABLE III

	$\frac{(\text{Contour})_1}{S_{ND}}$	$\frac{(\text{Contour})_0}{S_{ND}}$	$\frac{(\text{Contour})_0}{(\text{Contour})_1}$
0·0	1	1	1
·2	1	1	1
·4	1·01	0·99	0·98
·6	1·025	·975	·95
·8	1·05	·95	·90
·9	1·07	·93	·87

Table III shows that between $(\text{contour})_0$ and $(\text{contour})_1$ there would be a difference of 13 per cent. of absorption at $v/V = 0{\cdot}9$.

7. *Fourth Problem.—P proportional to g_{eff} ; N_1 and N_0 of the same order ; with consideration of darkening at the limb.*

The expression for $(\text{contour})_1$ is now

$$(\text{Contour})_1 = \int_{\theta_1}^{\frac{\pi}{2}} \left\{(1-u)\sin\theta + u\sin^2\theta \left[1 - \left(\frac{\nu - \nu_0}{\nu_0}\right)^2 \cdot \frac{c^2}{V^2\sin^2\theta}\right]^{\frac{1}{2}}\right\} \cdot \frac{K d\theta}{P_0(1 - \epsilon\sin^2\theta) + K}.$$

Let us write

$$a^2 = \frac{P_0\epsilon}{K + P_0 - P_0\epsilon}.$$

The expression obtained for $(\text{contour})_1$ is

$$(\text{Contour})_1 = (1-u) \cdot \frac{aK}{P_0\epsilon} \cdot \tan^{-1}(aG(\nu)) + \frac{\pi u K}{2P_0\epsilon}\left(\sqrt{1 + a^2G^2(\nu)} - 1\right);$$

or neglecting the constant factor $\frac{K}{P_0\epsilon}(\epsilon \neq 0)$

$$(\text{Contour})_1 = (1-u)\,.\,a\tan^{-1}(aG(\nu)) + \frac{\pi}{2}u(\sqrt{1+a^2G^2(\nu)} - 1). \qquad (11)$$

The numerical results are similar to those of the preceding tables.

8. *Fifth Problem.—P proportional to square root of g_{eff} ; no consideration of darkening at the limb ; K very large.*

The expressions (7′) have to be used ; when K is very large, $(\text{contour})_1$ is evidently $G(\nu)$. For $(\text{contour})_0$ we have the expression

$$(\text{Contour})_0 = \int_{\theta_1}^{\pi/2} \sqrt{1-\epsilon\sin^2\theta}\,\sin\theta\,d\theta\,;$$

this brings

$$(\text{Contour})_0 = \tfrac{1}{2}\left[G(\nu)\sqrt{1-\epsilon+\epsilon G^2(\nu)} + \frac{1-\epsilon}{\sqrt{\epsilon}}\,.\,\sinh^{-1}G(\nu)\sqrt{\frac{\epsilon}{1-\epsilon}}\right]. \qquad (12)$$

Of course, when ϵ is small, the expression (12) reduces to formula (8) where ϵ is replaced by $\epsilon/2$.

9. *Sixth Problem.—P proportional to square root of g_{eff} ; no consideration of darkening at the limb ; N_0 and N_1 of the same order.*

$(\text{Contour})_1$ is given by

$$(\text{Contour})_1 = \int_{\theta_1}^{\frac{\pi}{2}} \frac{K\sin\theta\,d\theta}{K + P_0\sqrt{1-\epsilon\sin^2\theta}}.$$

The expression obtained by integration is rather inconvenient for numerical calculations ; it is

$$(\text{Contour})_1 = \frac{1}{P_0\sqrt{\epsilon}}\left[\sinh^{-1}\left(G(\nu)\sqrt{\frac{\epsilon}{1-\epsilon}}\right) + \frac{K}{\sqrt{K^2 - P_0^2(1-\epsilon)}}\,.\,\log_e\frac{mn_0}{nm_0}\right], \qquad (13)$$

with following abbreviations :—

$$m = \sqrt{\epsilon}G + \sqrt{\epsilon G^2 + 1 - \epsilon} + \frac{K}{P_0} + \frac{1}{P_0}\sqrt{K^2 - P_0^2(1-\epsilon)}\,;$$

$$m_0 = m(G \to 0) = \sqrt{1-\epsilon} + \frac{K}{P_0} + \frac{1}{P_0}\sqrt{K^2 - P_0^2(1-\epsilon)}\,;$$

$$n = \sqrt{\epsilon}G + \sqrt{\epsilon G^2 + 1 - \epsilon} + \frac{K}{P_0} - \frac{1}{P_0}\sqrt{K^2 - P_0^2(1-\epsilon)}\,;$$

$$n_0 = n(G \to 0) = \sqrt{1-\epsilon} + \frac{K}{P_0} - \frac{1}{P_0}\sqrt{K^2 - P_0^2(1-\epsilon)}.$$

Similar formulæ may be obtained in case *b*, introducing the darkening towards the limb ; but the expressions are complicated and have no interest with regard to numerical applications.

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THE EFFECT OF SPACE REDDENING ON THE BALMER DECREMENT IN PLANETARY NEBULÆ.

Louis Berman.

(Communicated by Dr. R. J. Trumpler)

I. *Introduction*

In an interesting paper on the spectra of certain planetary nebulæ, T. L. Page * seconds a suggestion of S. Chandrasekhar that the observed Balmer line decrement is more rapid than that given by theory † owing to the effect of self-reversal in the nebulæ. It appears, however, that most of this discrepancy may be interpreted on the basis of space reddening which must affect the continuous nebular and nuclear emission as well as the Balmer gradient. That such a phenomenon exists was first pointed out by G. A. Shajn,‡ who observed that the line decrement steepens with decreasing galactic latitude.

In view of the immediate interest occasioned by this problem, the writer deems it pertinent at this time to bring forward some results of an unpublished investigation of the planetary nebulæ having a direct bearing upon the matter. The object of this paper is to present the observational evidence which appears to indicate that the variation in the hydrogen line decrement is a function of the effective path-length of the nebular light through an absorbing medium in space.

II. *Cosmic Absorption*

The individual distances of the nebulæ have been computed by the writer from a study of the galactic rotation based upon the radial velocities, proper motions, apparent magnitudes and apparent diameters of the planetaries.§ From the same data the photographic absorption in the galactic plane is found to be 0·7 mag. per kiloparsec, corresponding to a "400-parsec" medium of uniform thickness having an optical depth of 0·3 mag. As a working approximation to the probable actual conditions, we shall assume that the density in this absorbing stratum perpendicular to the galactic equator varies according to the formula ||

$$\rho = \rho_0 e^{-\pi\left(\frac{z}{z_0}\right)^2}, \qquad (1)$$

* T. L. Page, *M.N.*, **96**, 604, 1936.

† G. Cillié, *M.N.*, **92**, 820, 1932.

‡ G. A. Shajn, *Poulkovo Obs. Circ.*, No. 11, 8, 1934.

§ This investigation is now being prepared for publication. Owing to significant differences in method, our distances are generally greater than those found by B. Vorontsov-Velyaminov, *Zs. f. Astrophysik*, **8**, 14, 1934.

|| Suggested to the writer by Dr. R. J. Trumpler.

where ρ_0 represents the density in the galactic plane, z the vertical distance from this plane and z_0 the total effective thickness (400 parsecs). Hence, it follows that the space absorption, expressed in magnitudes at any wave-length, λ, is given by

$$\Delta m_\lambda = k_\lambda z_0 \operatorname{cosec} b \frac{1}{\sqrt{\pi}} \int_0^x e^{-x^2} dx = k_\lambda r_1, \quad \text{where} \quad x = \frac{\sqrt{\pi} r \sin b}{z_0}. \qquad (2)$$

Here, k_λ is the coefficient of interstellar absorption at λ (that is, its absolute value per thousand parsecs), b the galactic latitude of an object, r its distance from the Sun and r_1 its effective path-length in the medium. With the given constant, z_0, one may calculate r_1 corresponding to any distance, r, and galactic latitude, b, and the amount of absorption, provided k_λ is known.

III. *The Spectrophotometric Data*

The nebular line intensities have been assembled from the following sources : (1) H. H. Plaskett, *Publ. D.A.O.* (*Victoria*), **4**, 121, 1928 [8 objects] ; (2) T. L. Page, *M.N.*, **96**, 124, 1936 [3 objects] ; (3) W. H. Wright, *Publ. Lick Obs.*, **13**, 193, 1918 [36 objects] ; (4) L. Berman, *Lick Obs. Bull.*, **15**, 86, 1930 [5 objects], to which the results from 14 additional planetaries, observed in 1932, are now added.*

Plaskett's slit spectra were secured at the Cassegrain focus of the 72-inch Victoria reflector with the one-prism form of the universal spectrograph. His plates were photometrically standardized by means of a wedge method and the total line intensities were reduced to an energy scale by calibration with respect to the known intensity distribution of either one of two sources, a carbon arc or an Ao star.

Page's intensities, which depend upon wide slit spectra taken by Plaskett in 1930 with an ultra-violet glass spectrograph mounted at the principal focus of the Victoria reflector, were reduced to absolute values by methods essentially similar to those previously employed by Plaskett.

Both Wright's and Berman's spectrograms were obtained with the same apparatus : the quartz two-prism slitless spectrograph attached to the 36-inch Crossley reflector at the prime focus. In addition, the latter also employed the slit adaption of the same instrument in 1932. The relative approximate photographic intensities of nebular lines published by Wright † in his Table XIII have been reduced to a true energy scale by applying the sensitivity curve of Seed 23 plates as determined by Yü.‡ The nebular densities were originally measured against a calibrated wedge on the Hartman microphotometer. Despite the lack of contrast standardization on these older plates, the resulting intensities are in reasonable agreement with the writer's later determinations, and their inclusion on this account, in a capacity for which they were not originally intended, appears to be justified.

* A detailed account of this work, including the observational details and reduction methods, will appear in the future.

† W. H. Wright, *Publ. Lick Obs.*, **13**, 246, 1918.

‡ C. Yü, *Lick Obs. Bull.*, **12**, 120, 1926.

TABLE I

Mean Hydrogen Intensities

Object	Type	r	r_1	N_2	$H\beta$	$H\gamma$	$H\delta$	$H\epsilon$	$H\zeta$	$H\eta$
					Group I					
N.G.C. 6790	IIa	4·30	1·44	−1·73 (1)	0·00 (2)	+0·99	+2·02	+2·75	+2·60 (1)	+2·90
N.G.C. 2440	Ia	2·44	2·05	1·69 (2)	0·00	0·98	2·00 (1)	3·05		
N.G.C. 6720	Ib	1·52	0·86	1·65 (1)	0·00	1·05	1·80	2·75		
N.G.C. 6884	Ic	6·08	1·64	1·62 (2)	0·00	1·04	1·96	2·60	3·05	3·70
N.G.C. 6741	Ia	3·53	2·52	1·59 (2)	0·00	1·34	2·23	3·00	3·40	
N.G.C. 7027	Ia	1·85	1·67	1·58 (4)	0·00 (5)	1·14	2·21	2·70 (4)	3·15 (3)	3·45
N.G.C. 6886	Ia	6·73	1·28	1·58 (2)	0·00	1·12	1·93	2·45	2·75 (1)	2·90
N.G.C. 7662	Ib	1·18	0·66	−1·43 (2)	0·00 (3)	+0·96	+1·74	+2·45 (2)	+2·75 (1)	+3·25
					Group II					
I.C. 5117	IIa	6·34	1·92	−1·36 (2)	0·00	+1·24	+2·18	+2·90	+3·40	
N.G.C. 3242	Ib	1·60	0·37	1·31 (2)	0·00	0·88	1·63	2·35	2·75 (1)	
N.G.C. 1747	IIa	3·68	3·32	1·26 (1)	0·00	[0·85]	tr.*			
N.G.C. 6818	Ia	2·27	0·59	1·23 (1)	0·00	0·88	1·86	2·50		
N.G.C. 7009	Ib	0·90	0·35	1·22 (3)	0·00	0·89	1·64	2·20	2·75 (2)	+3·25
I.C. 351	Ib	5·13	0·77	1·20 (1)	0·00	0·95	1·80	2·15	2·50	2·75
N.G.C. 1535	Ic	1·74	0·32	1·19 (1)	0·00	0·92	1·75	2·40		
N.G.C. 7026	Ic	2·75	2·75	1·18 (1)	0·00	1·15	[1·75]	[2·60]		
I.C. 2165	Ib	4·49	1·05	1·18 (1)	0·00	0·95	1·80	2·50	3·05	3·50
N.G.C. 6803	IIa	2·39	1·73	1·10 (1)	0·00	0·93	1·80	2·60		
I.C. 3568	IIa	2·33	0·21	1·05 (2)	0·00 (3)	0·74	1·56	2·15	2·50 (2)	
N.G.C. 2392	Ia	0·85	0·55	1·00 (1)	0·00	0·85	1·55	2·40		
I.C. 5217	Ib	2·91	1·86	0·99 (2)	0·00	1·03	1·82	2·45	2·90	3·40
N.G.C. 6891	IIb	2·92	0·88	−0·98 (2)	0·00	+1·01	+1·71	+2·40	+2·90 (1)	+3·75
					Group III					
N.G.C. 6572	IIa	1·14	0·90	−0·95 (3)	0·00	+0·88	+1·58	+2·00	+2·30	+2·75
I.C. 4634	IIa	3·22	1·05	0·95 (1)	0·00	0·82	1·64	2·20	2·75	3·25
N.G.C. 6210	IIa	1·85	0·17	0·93 (1)	0·00 (2)	0·76	1·52	1·95	2·45	3·10
I.C. 4846	IIa	6·34	1·05	0·90 (1)	0·00	0·77	1·55	2·10		
I.C. 4776	IIa	5·68	0·77	0·82 (1)	0·00	0·95	1·69	2·20	2·75	3·05
N.G.C. 6807	IIa	11·43	1·44	0·77 (1)	0·00	1·05	1·92	2·50	3·05	3·50
N.G.C. 6826	IIa	1·00	0·79	0·76 (1)	0·00 (2)	0·88	1·48	2·00	2·30 (1)	2·55
N.G.C. 6879	IIa	6·76	1·14	0·75 (1)	0·00	1·07	2·13	2·50		
I.C. 4593	IIb	2·02	0·31	0·58 (3)	0·00	0·79	1·55	2·05	2·50 (2)	2·80 (1)
N.G.C. 6543	IIa	1·05	0·40	0·54 (2)	0·00	0·85	1·52	2·00	2·40	2·70
N.G.C. 6833	IIa	9·38	1·05	0·50 (1)	0·00	0·78	1·64	2·30	2·90	3·50
I.C. 2149	IIb	2·08	0·96	0·38 (1)	0·00	0·85	1·30	2·00	2·40	2·75
N.G.C. 1976		0·54†	0·45	0·26 (1)	0·00	0·80	1·80			
I.C. 4997	IIa	2·13	0·96	−0·10 (2)	0·00 (3)	0·73	1·65	1·90 (2)	2·35	2·65
N.G.C. 418	III	1·68	0·56	+0·51 (1)	0·00	0·88	1·55	2·00	2·15	2·25
B.D. +30 : 3639	III	1·01	0·98	+1·8‡ (1)	0·00	1·00	2·00	2·60	3·00	+3·50
N.G.C. 40	III	1·14	0·90	tr.* (1)	0·00	+0·88	+1·75	+2·30	+2·75	

* Trace.

† According to R. J. Trumpler's determination for the Orion Nebula, *Publ. Astr. Soc. Pac.*, 43, 255, 1931.

‡ N_2 intensity estimated from Wright's glass-prism spectra, *loc. cit.*, p. 242.

From more than 50 slit and slitless spectrograms secured during the summer of 1932 on Imperial Eclipse Plates, the writer has derived the total integrated or line intensities of 14 nebulæ of which only two are common to those observed by him in 1928. The photometric procedure relating to standardization and calibration of the plates is similar to the one described in a previous investigation,* with the exception that in place of the Sun the star ζ Ophiuchi (Bo) with an assumed temperature of 22,000° K was adopted as the standard source of intensity distribution.

Table I presents the observational data upon which the present investigation is based. To eliminate any dependence of the nebulium intensities on the Balmer line gradient, the nebulæ have been divided into three groups according to the brightness of the principal nebular lines, N_1 and N_2. The first column of the table gives the designated object as listed in the New General or Index Catalogues; the second column, the type as classified by Wright; the third and fourth columns, the computed distances, r, and effective distances in the "400-parsec" absorbing medium, r_1, given in kiloparsecs from the Sun; and the remaining columns, the mean nebular intensities, expressed in magnitudes on a true energy scale, and arbitrarily referred to $H\beta$ as the zero point.† The number of observations entering into the averages is shown by the figure enclosed in parentheses. Where no figure is indicated, it will be understood that the number of determinations is the same as the last preceding figure.

The values for $H\epsilon$ and $H\zeta$ were obtained by direct interpolation of the Balmer intensities whenever $H\eta$ was present; otherwise, in the case of $H\epsilon$, by subtracting the intensity of λ 3967 (assuming the intensity ratio λ 3869 : λ 3967 = 3·4 : 1 derived from those objects for which an interpolated intensity of $H\epsilon$ was available) and by extrapolation from $H\delta$. The intensities for the last three hydrogen lines are given only to the nearest 0·05 magnitude.

IV. *The Law of Selective Absorption*

The effect of space reddening upon the Balmer line gradient is exhibited graphically for Group I in fig. 1. Two phenomena may be noticed, viz. for a given wave-length relative to $H\beta$ the lines become progressively fainter, due to increased space absorption, and for different wave-lengths the slope of the line intensities increases towards the ultra-violet owing to the selective nature of the absorption.

From the data of Table I we may determine the law of selective absorption

* L. Berman, *Lick Obs. Bull.*, **15**, 86, 1930. The intensities derived in this paper require a systematic correction depending upon the slope of the dispersion of the slitless spectrograms. This factor, which was inadvertently omitted, has now been applied to the intensities so that the results are in much better agreement with those of other observers. For some of the principal lines the correction, expressed in magnitudes and to be added to the intensities, is as follows: N_1, −0·09; $H\beta$, 0·00; $H\gamma$, +0·38; $H\delta$, +0·58; $H\epsilon$, +0·72; $H\zeta$, +0·80; $H\eta$, +0·85; λ 3869, +0·83; λ 3727, 1·00.

† Of the nebulium pair, only N_2 is listed, since it bears a definite intensity ratio to N_1.

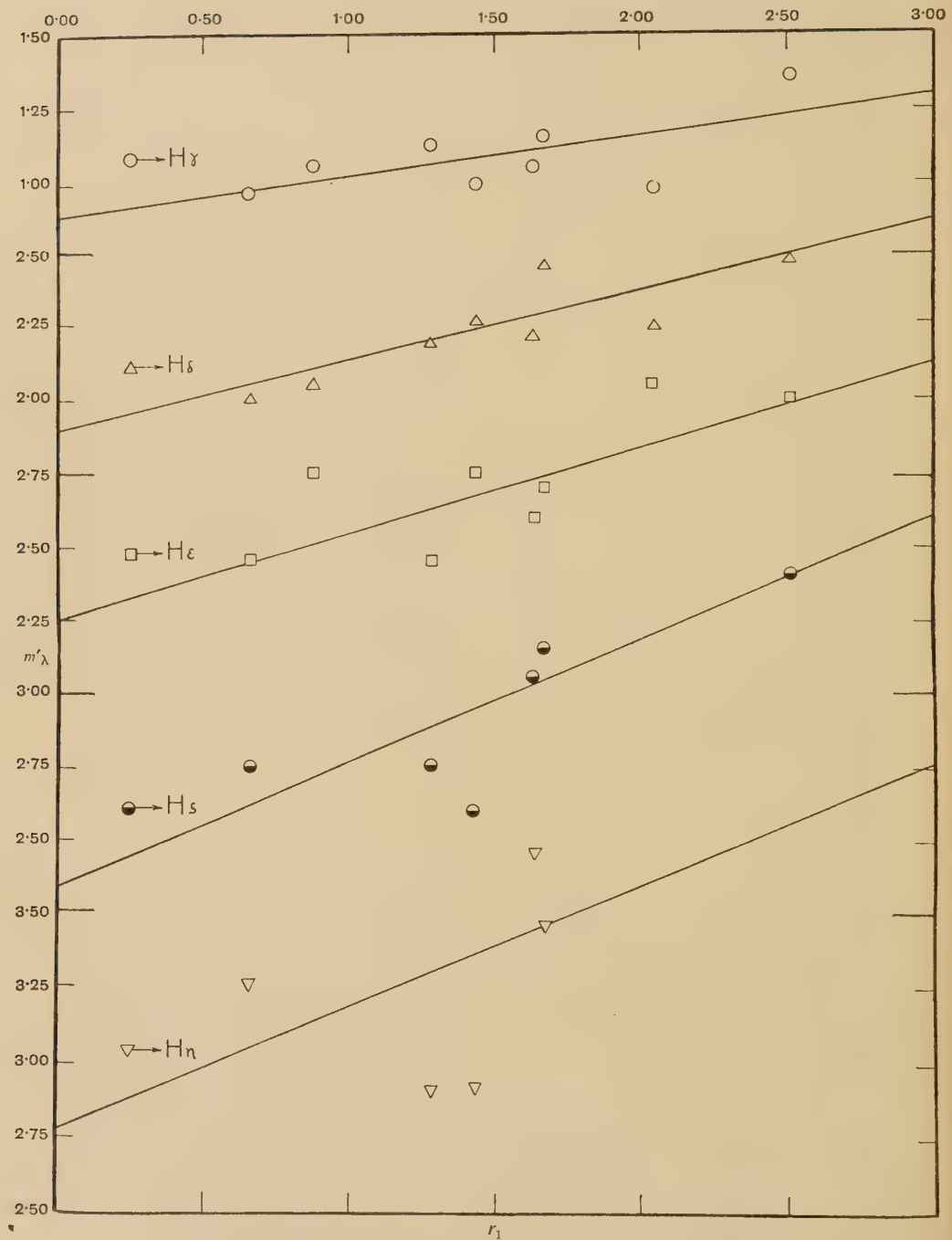

FIG. 1.—*The Effect of Space Reddening upon the Hydrogen Intensities.*

and the true Balmer decrement unaffected by absorption. Let c_λ be the colour excess in a line, λ, relative to $H\beta$ per thousand parsecs. In our notation, $c_\lambda = k_\lambda - k_{H\beta}$. If m_λ' is the observed magnitude of λ and m_λ the magnitude undisturbed by absorption, we have the simple observation equation

$$m_\lambda' - c_{\lambda 1} = m_\lambda \qquad (3)$$

in which c_λ and m_λ are the two unknowns to be solved by the method of least squares. In Group II, three values, indicated by enclosed brackets, were rejected as falling considerably below the curve for the others.* With a weight assignment depending upon the number of individual determinations, three independent solutions of c_λ and m_λ corresponding to each of the three groups, I, II and III, were obtained.

The numerical results, expressed in magnitudes, together with the probable errors, are tabulated in Table II.

TABLE II

Colour Excess (c_λ) *and Balmer Intensities* (m_λ) (*in Magnitudes*)

	Group	c_λ		m_λ	
	I	+0·136 ±	0·025	+0·875 ±	0·042
4341 ($H\gamma$)	II	0·158	0·013	0·798	0·016
	III	0·125	0·039	0·749	0·032
	I	0·223	0·036	1·678	0·060
4102 ($H\delta$)	II	0·225	0·025	1·554	0·026
	III	0·234	0·066	1·453	0·055
	I	0·290	0·037	2·248	0·060
3970 ($H\epsilon$)	II	0·278	0·031	2·157	0·032
	III	0·316	0·070	1·867	0·058
	I	0·418	0·062	2·347	0·108
3889 ($H\zeta$)	II	0·344	0·054	2·522	0·065
	III	0·345	0·103	2·261	0·084
	I	0·409	0·215	2·775	0·375
3835 ($H\eta$)	II	0·127	0·147	3·199	0·173
	III	+0·366 ±	0·165	+2·627 ±	0·136

The final values of the colour excess, referred to $H\beta$, were then determined for each line by weighting the individual c_λ according to their probable errors. They are presented in Table III together with the number of observations upon which they depend, placed in parentheses.

TABLE III

Final Values of the Colour Excess (c_λ) (*in Magnitudes*)

$H\beta$	$H\gamma$	$H\delta$	$H\epsilon$	$H\zeta$	$H\eta$
0·000 (65)	+0·151 (65)	+0·225 (64)	+0·287 (60)	+0·372 (42)	+0·270 (33)

* One of the nebulæ involved, N.G.C. 7026, which is fairly bright, according to H. D. Curtis (*Publ. Lick Obs.*, 13, plate xxii., 1918), lies in the galactic plane, and it is possible that the intensities are too high, because there may be less obstruction than usual in this region ($l = 57°$).

To establish the law of selective absorption—that is, the numerical value of the exponent, n—in the expression, $k_\lambda \propto \lambda^{-n}$, we put it in the form

$$k_\lambda = k_{H\beta}\left(\frac{4862}{\lambda}\right)^n, \tag{4}$$

where $k_{H\beta}$ is the absorption constant for $H\beta$. Since the present data are insufficient to determine $k_{H\beta}$, its value may be fixed relative to the known photographic absorption constant. Hence, employing the results of Table III, we find $k_{H\beta} = 0{\cdot}53$ mag. derived from the known coefficient of photographic absorption, 0·70 mag. (λ 4300) by successive approximations with steadily improved values of n. With this value of $k_{H\beta}$ we obtain by a least square reduction, after having assigned weights based on the total number of observations entering into each determination of c_λ as indicated in the last table, the final value of $n = 2{\cdot}27 \pm 0{\cdot}08$. Had we adopted the value of 1·00 mag. (probably an upper limit) for the photographic constant, $k_{H\beta}$ would be 0·83 mag. and $n = 1{\cdot}49 \pm 0{\cdot}06$. It is evident that the exponent must have a value very close to 2, or about half that required by the Rayleigh scattering formula. Either set of constants, determined above, gives for the differential absorption between the visual (λ 5600) and the photographic (λ 4300) regions the value 0·32 mag., in good agreement with other determinations.*

V. *Correction of Intensities*

It is now possible to correct Page's intensities of the continuous spectra in three of the nebulæ observed by him.† Table IV presents his measures referred to λ 3927 as the intensity unit and the same quantities corrected for space reddening by applying the colour excess, $c_\lambda = 0{\cdot}86\left[1 - \left(\frac{3927}{\lambda}\right)^{2\cdot27}\right] r_1$ where 0·86 mag. is the absorption constant for λ 3927.‡

The improved intensities thus definitely strengthen Page's conclusion that the nebular matter which scatters the visible nuclear light and gives rise to the continuous nebular emission behaves like grey material. The net effect of the selective absorption is to increase the amount of Balmer continuous emission (relative to the hydrogen lines) which he observed without appreciably altering its quality, and to double his observed colour temperature of 16,000° K for the nucleus of N.G.C. 7662.§

The high intensities of $H\alpha$ found by Plaskett || in N.G.C. 1976 and in

* G. Shajn, *Zs. f. Astrophys.*, **8**, 178, 1934.

† T. L. Page, *loc. cit.*, p. 617.

‡ It is immaterial which set of constants, $k_{H\beta}$ and n, is employed in calculating c_λ, since they give practically the same results by compensating one another. We have used $k_{H\beta} = 0{\cdot}53$ magnitude and $n = 2{\cdot}27$.

§ Owing to the small spectral range ($\lambda\lambda$ 3480–3680), the low electron temperatures of about 2000° K which Page derived from the intensity distribution of the Balmer continuous emission are not materially affected. Incidentally, the colour temperatures of 25,000°–30,000° K found by the writer for the nuclei of three planetaries should be increased by at least 50 per cent. (*Lick Obs. Bull.*, **15**, 99, 1930).

|| H. H. Plaskett, *Publ. D.A.O.* (*Victoria*), **4**, 121, 1928.

TABLE IV

Corrected Continuous Emission Intensities

λ	N.G.C. 7662 (4″·9 West of Nucleus)		N.G.C. 7662 (4″·9 East of Nucleus)		N.G.C. 7027		N.G.C. 6572	
	Observed	Corrected	Observed	Corrected	Observed	Corrected	Observed	Corrected
4800	1·23	1·01	1·34	1·11	2·00	1·23	1·12	0·86
4416	0·86	0·77	1·11	0·98	1·23	0·90	1·19	1·01
4270	0·95	0·87	1·08	0·98	1·20	0·95	1·20	1·06
4152	0·92	0·87	0·99	0·94	1·03	0·89	1·11	1·02
4062	1·01	0·97	1·09	1·05				
3927	1·00	1·00	1·00	1·00	1·00	1·00	1·00	1·00

N.G.C. 7027, particularly, are now seen to be due in large measure to the effect of space reddening. Table V reproduces his observed intensity ratio, $H\alpha/H\beta$, together with the results derived for two other nebulæ by the writer. The intensities corrected for absorption appear in adjoining columns. The writer's results rest upon one slitless spectrogram of B.D. +30 : 3639 and two of N.G.C. 6210 taken with Eastman Hypersensitive Plates. The great difference in the apparent photographic intensity between $H\alpha$ and $H\beta$ on the plates, coupled with the strong likelihood that the standard star, ζ Ophiuchi, may be slightly affected by selective absorption * (which tends to decrease the ratio $H\alpha/H\beta$), add to the uncertainty of measurement.

TABLE V

Corrected Hα Intensities

	N.G.C. 7027		N.G.C. 1976		B.D. +30 : 3639		N.G.C. 6210	
	Observed	Corrected	Observed	Corrected	Observed	Corrected	Observed	Corrected
$H\alpha/H\beta$	5·8	3·9	4·0	3·5	3·1	2·4	2·5	2·4

VI. *The Balmer Decrement*

Table VI summarizes the hydrogen intensity data according to the three groups into which the nebulæ were arranged. All quantities are expressed in terms of $H\beta$ as the arbitrary unit of intensity.

The N_2 intensities are taken from Table I and the $H\alpha$ intensities from

* J. Stebbins and C. M. Huffer find a colour excess index of +0·11 magnitude (*Publ. Washburn Obs.*, 15, Part V, 232, 1934).

TABLE VI

Balmer Line Decrement

Atom	Group I, 8 Nebulæ	Group II, 13 Nebulæ	Group III, 17 Nebulæ	Theoretical Balmer Decrement			Thermodynamic Equilibrium
	$T_\epsilon = 1000°$ *	...	$2500°$ *	$1000°$	$5000°$	$50{,}000°$	$2500°$
4959 $O_{III}(N_2)$	4·37	2·91	1·81†				
6563 $H\alpha$	3·9	...	2·77	2·61	2·82	3·08	44·5
4862 $H\beta$	1·00	1·00	1·00	1·00	1·00	1·00	1·00
4341 $H\gamma$	0·45	0·48	0·50	0·50	0·47	0·44	0·13
4102 $H\delta$	0·21	0·24	0·26				0·035
3970 $H\epsilon$	0·13	0·14	0·18				0·014
3889 $H\zeta$	0·11‡	0·10	0·12				0·007
3835 $H\eta$	0·08‡	0·05	0·09				0·004
	§ $T_s = 65{,}000°$	$47{,}000°$	$38{,}000°$				

* Observed electron temperatures according to Page (*loc. cit.*, p. 626).

† Excluding Type III from the mean.

‡ These values are of low weight.

§ Nuclear temperatures, T_s, according to calculations based on R. H. Stoy's method (*M.N.*, **93**, 588, 1933) from the ratio of nebulium to hydrogen intensities.

Table V; the remaining observed hydrogen intensities from the fourth column of Table II. Columns 6, 7 and 8 contain the predicted Balmer decrement calculated from formulæ due to Cillié,* who derived expressions for the number of electron captures on the hydrogen levels as a function of the electron temperature, $T\epsilon$. For comparison, the decrement predicted on the assumption of thermodynamic equilibrium ($T_\epsilon = 2500°$ K) is presented in the final column.

Thus, the variations which Cillié found to exist from nebula to nebula between the observed and theoretical intensities of $H\alpha/H\beta$ and $H\gamma/H\beta$ find an almost complete explanation in the phenomenon of space reddening. There remains, however, a small residual effect, undoubtedly real, noticeable in the observed decrement of Group I, in a sense contrary to theory. For example, the intensity distribution of the Balmer continuous spectrum observed in four nebulæ by Page indicates that the electron temperatures decrease with higher N_2 intensity (or nuclear temperature). The cooling of the electron gas results, of course, from the circumstance that the free electrons dissipate most of their kinetic energy by collisions of the first kind with nebulium atoms before they are slowed up sufficiently to recombine with protons. Now a comparison between the observed and predicted decrement does, indeed, show that the electron temperatures are extraordinarily low, as Page observed ($\sim 2000°$), but for the lower temperatures the observed decrement is more rapid than that given by theory (Group I). In order to account for this discrepancy, Page suggested that the small expected flattening of the hydrogen line decrement due to decreased T_ϵ is overcome by the reverse effect due to self-absorption which increases with the nuclear temperature so that the decrement is accelerated. Although his intensities reveal a more rapid decrement, since they are influenced by

* G. Cillié, *M.N.*, **92**, 825, 1932; T. L. Page, *M.N.*, **96**, 626, 1936.

selective absorption, it is now apparent that the difference between theory and observation has been reduced to one of a minor nature.

It is of interest to ascertain the deviation of the relative partition of hydrogen atoms among the various states from thermodynamic equilibrium. Assuming a Boltzmann distribution relative to the second level ($n=2$), one derives the expression

$$\text{Log } D = \log\left(\frac{2}{n}\right)^2 \frac{I_{n2}}{N_2 A_{n2} h\nu_{n2}} = -\frac{5040(E_n - E_2)}{T} + C, \qquad (4)$$

where I_{n2} is the observed Balmer line emission, $E_n - E_2$, the energy difference, in volts, between the higher and second levels, C is an arbitrary constant, and the other terms have their usual meanings. If the intensities are plotted against $E_n - E_2$ according to the above formula, a fairly smooth exponential decrease is indicated. Interpreted in the usual way, the slope ($= -5040/T$) corresponds to an "excitation temperature" of 25,000° on the basis of thermodynamic equilibrium. This absurdly high value results from the circumstance that the relative defection of atoms in the various states as compared with thermodynamic equilibrium becomes very pronounced, especially in the lower levels.

VII. *Summary*

From the distances of the planetary nebulæ and an effective width of 400 parsecs for the absorbing layer, it is shown that (1) the observed hydrogen line intensities grow progressively fainter with increasing path-length of the nebular light through the absorbing medium (general absorption), and (2) the weakening of the lines is accelerated toward the ultra-violet (selective absorption). From the observed data, the coefficient of absorption, relative to Hβ, is found to be $k_\lambda = 0{\cdot}53\left(\frac{4862}{\lambda}\right)^{2{\cdot}27}$, where 0·53 is the absorption constant at Hβ, expressed in magnitudes, per kiloparsec. Except for a small difference which may be attributed to self-reversal, the observed Balmer line decrement, corrected for absorption, is in satisfactory accord with theory.

VIII. *Acknowledgments*

It is a pleasure to acknowledge the assistance of Director W. H. Wright and Dr. R. J. Trumpler, of the Lick Observatory, to whom the writer is indebted for a number of suggestions which have been incorporated in the manuscript.

San Francisco Junior College,
San Francisco, California :
August 1936.

Addendum

In a paper which appeared after the present one was completed, G. Cillié (*M.N.*, **96**, 771, 1936) has extended his earlier computations on the theoretical hydrogen decrement from $H\alpha$ to $H\theta$. A comparison of the present observed decrement with that predicted by Cillié, on the basis of Page's observed electron temperatures, still reveals a small discrepancy which may be attributable to self-reversal in the lines.

OCCULTATIONS OF STARS BY THE MOON, OBSERVED AT THE NIZAMIAH OBSERVATORY, HYDERABAD, DURING THE YEAR 1935.

(Communicated by the Director)

No.	Star	Mag.	Phase	Date	U.T.
		m		1935	h m s
1	4 Arietis	5·7	D	Jan. 12	13 39 32·1
2	Paris ph. +21° 8^h 4^m 379	10·4	D	19	15 0 17·2
3	Do. do. 370	9·6	D	19	15 1 34·0
4	Do. do. 396	10·2	D	19	15 27 15·8
5	Do. do. 397	10·6	D	19	15 28 13·7
6	Do. do. 389	9·9	D	19	15 28 44·6
7	Do. do. 394	10·1	D	19	15 39 0·9
8	Do. do. 400	9·6	D	19	15 47 28·6
9	Do. do. 414	10·2	D	19	15 49 30·7
10	Do. do. 402	10·1	D	19	16 3 2·0
11	Do. do. 433	10·6	D	19	16 12 21·5
12	Do. do. 427	10·2	D	19	16 19 43·9
13	η Piscium	3·7	D	Feb. 8	13 54 16·3
14	Abbadia 4268	8·4	D	Apr. 10	14 5 35·0
15	58 Geminorum	6·0	D	10	14 43 28·2
16	„ „	6·0	R	10	15 46 22·9

All the observations were made by T. P. Bhaskaran with the exception of Nos. 1, 14, 15 and 16, which were made by M. K. Bappu. The instrument used was the 15-inch Grubb refractor and the times in all cases were noted with a stop-watch ; the clock corrections were determined from observations with the transit instrument.

Remarks : Nos. 2 to 12 were observed during the lunar eclipse of January 19, 1935. No. 16—the time is uncertain, the observation being slightly late, as the star appeared very faint.

The reductions have been carried out by M. V. V. Sastri and S. Aravamudan. A correction of $0^h{\cdot}0025$ has been added to the U.T. to allow for the error of 5″ in the mean longitude of the Moon.

No.	χ	ρ	$\sigma' - \sigma$	$\cos(\chi - \rho)$
	° ′	° ′	″	
1	46 31	68 52	− 2·6	+0·925
2	83 47	106 55	+ 4·0	+0·920
3	49 54	106 56	+ 3·0	+0·544
4	73 49	107 0	+ 4·7	+0·837
5	69 53	107 1	+ 0·9	+0·797
6	51 25	107 1	+ 2·8	+0·565
7	42·22	107 3	+ 2·8	+0·428
8	131 13	107 4	− 1·4	+0·912
9	96 35	107 4	− 1·8	+0·983
10	26 54	107 7	+ 3·2	+0·170

No.	χ	ρ	$\sigma' - \sigma$	$\cos(\chi - \rho)$
	° ′	° ′	″	
11	112 19	107 9	+ 0·3	+0·996
12	141 57	107 10	+ 1·3	+0·821
13	72 19	67 53	− 4·1	+0·997
14	53 40	102 43	− 1·0	+0·655
15	74 20	102 51	− 2·1	+0·879
16	341 5	103 4	+10·1	−0·530

Nizamiah Observatory,
Hyderabad, India.

OCCULTATIONS OF STARS BY THE MOON, OBSERVED AT ENGELHARDT OBSERVATORY, KASAN, DURING THE YEAR 1935.

No.	Date 1935	U.T.	Star	Mag.	Phase	Instrument and Power	Observer
		h m s					
1	Jan. 14	17 54 31·7	17 Tauri	3·8	D	R 143	T
2	14	18 0 36·4	16 Tauri	5·4	D	R 143	T
3	14	18 39 48·0	20 Tauri	4·1	D	R 143	T
4	14	18 39 47·8	,,	,,	D	M 56	M
5	14	19 20 6·1	B.D. 23°540	7·0	D	R 143	T
6	14	19 28 11·1	η Tauri	2·9	D	R 143	T
7	14	19 28 11·1	,,	,,	D	M 56	M
8	14	20 40 57·5	27 Tauri	3·8	D	R 143	T
9	Oct. 8	18 21 48·6	96 B Aquar.	6·5	D	H 132	B
10	8	18 21 48·3	,,	,,	D	A 110	N
11	14	23 49 43·9	104 B Tauri	5·5	D	H 132	B
12	Nov. 6	19 38 23·1	22 B Piscium	6·5	D	H 132	B
13	6	19 38 22·0	,,	,,	D	A 110	N
14	6	19 38 24·5	,,	,,	D	Z 64	T-a
15	6	19 38 23·0	,,	,,	D	Fr 108	T
16	6	21 50 41·5	9 Piscium	6·4	D	H 132	*B*
17	6	21 50 41·5	,,	,,	D	M 56	M
18	6	21 50 40·7	,,	,,	D	A 80	T
19	6	21 50 41·0	,,	,,	D	*Z* 64	T-a

The reductions were made by I. W. Belkowitsch.

The time-corrections of the chronometer were found from the Rugby wireless signals.

Observers :

B = I. W. Belkowitsch.
M = D. I. Martinoff.
N = Miss S. V. Nekrasova.
T-a = Mrs. N. A. Tchudowitcheva.
T = N. I. Tchudovitchev.

Instruments used :

A = Astrograph (110 mm.).
H = Heliometer (106 mm.).
R = Refractor (306 mm.).
M = Refractor (150 mm.).
Fr = Fraunhofer telescope (98 mm.).
Z = Zeiss telescope (68 mm.).

Engelhardt Observatory :
1936 *August.*

Errata

R. O. Redman, "Photographic Photometry of the Elliptical Nebulæ," *M.N.*, **96**, 602–3, Table IV, 5, 6.—The distribution figures for N.G.C. 3607 and 3608 should be interchanged.

S. W. Shiveshwarkar, "Remarks on Some Theorems in the Dynamics of a Steady Stellar System," *M.N.*, **96**, p. 750, line 18, *for* K *read* k ; p. 757, line 9, *for* $\sqrt{1+\frac{A}{\varpi}}$ *read* $\sqrt{1+\frac{A}{\omega}}$.

LIST OF ADDITIONS

TO THE

LIBRARY OF THE SOCIETY

1935 AUGUST 1 TO 1936 JULY 31

Serial Publications received by exchange in the regular course, and Reprints of Papers published in some of these Serials, are not included in this Annual List.

Unless otherwise indicated, the donor is the author or the Institution concerned.

An asterisk (*) *indicates that the work is an excerpt.*
A dagger (†) *indicates that the work has been purchased.*

Aitken (Robert Grant) :
†The Binary Stars. Second Edition. 8vo. New York and London, 1935

Algiers, Observatoire :
——: Catalogue fondamental de 872 étoiles rapporté à l'équinoxe moyen de 1925·0. 4to. Orléans, 1935
——: *Travaux. Vol. I, pp. 1–42 ; 55–114. 4to. Marseille, 1932–35

†Amateur Telescope Making :
(A collective work.) A. G. Ingalls, Editor. With a foreword by Dr. Harlow Shapley. Fourth Edition. sm. 4to. New York, 1935

Arcetri, Reale Osservatorio Astrofisico :
Immagini Spettroscopiche del Bordo Solare osservate a Catania, Dubossari, Madrid, Zurigo, 1929–1930.
(*International Astronomical Union.*) 4to. Firenze, 1935

Astronomische Nachrichten :
†General-Register der Bände 211 bis 240. 4to. Kiel, 1935

Atumi (K.) :
*La déviation de la verticale au Japon. 4to. *s.l.*, 1933

Barton (Samuel G.) :
*The Greatest Star Catalogue. (Science Service Radio Talks, presented over the Columbia Broadcasting System.) 8vo. *s.l.*, 1934

Belgrade, Observatoire Astronomique de l'Université :
——: Nouvelles tables de précession, par V. V. Michkovitch. 8vo. Beograd, 1935
——: Bulletin. Vol. I, Nos. 1, 2. 8vo. Beograd, 1936

Berlin, Preussische Akademie der Wissenschaften :
†Geschichte des Fixsternhimmels. Abteilung I. Der nördliche Sternhimmel. Bände 22, 23. 4to. Karlsruhe, 1935–36

Band 22. 21^h Rektaszension.
Band 23. 22^h Rektaszension.

Berlin-Babelsberg, Universitätssternwarte :
Veröffentlichung. Geschichte und Literatur des Lichtwechsels der veränderlichen Sterne. Zweite Ausgabe, enthaltend die Literatur der Jahre 1916–1933, von R. Prager. Band II. 4to. Berlin, 1936

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Astronomisch-geodätische Arbeiten in der Schweiz. Band 21. 4to. Bern, 1936

Band 21. P. Engi, Bestimmung von Längenunterschieden erster Ordnung mit drahtloser Uhrvergleichung in den Jahren 1924–1930.

Bernard (Henri) :
†Matteo Ricci's Scientific Contribution to China. Translated by Edward Chalmers Werner. 8vo. Peiping, 1935

Blagg (Mary A.) and K. Müller :
Named Lunar Formations. *See* **International Astronomical Union.**

Bordeaux, Société Astronomique de Bordeaux :
Albert Nodon: In Memoriam. 8vo. Bordeaux, 1935

Brisbane, Bureau of Seasonal Forecasting :
Seasonal Forecasting. Meteorology as a Branch of Astronomy. By Inigo Jones. 1930, 1931, 1932. 8vo. Brisbane, 1930–32

British Association for the Advancement of Science :
——: Catalogue of Earthquakes 1925–1930, being a Digest of the International Seismological Survey (1925–1930). By E. F. Bellamy. 8vo. London, 1935
——: Mathematical Tables. Vol. V. 4to. London, 1935

Vol. V. Factor Table giving the complete decomposition of all numbers less than 100,000.

(*B.A. Tables Committee.*)

Budapest-Svabhegy, Hungarian Astronomical Observatory :
Astronomical Circular, No. 1. 8vo. Budapest, 1935

Cartuja (Granada), Observatorio Geofisico :
Resumen del Boletin Sismico. (Edición aparte.) Nos. 1, 2. Año 1935. sm. 4to. Granada, 1936

Chapman (S.) :
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Chatley (Herbert) :
——: *Further notes on the history of the days of the week. 8vo. *s.l.*, 1933
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No. 21. Paul Herget, A method for determining preliminary orbits adapted to machine computation. Improved orbit of Biarmia 1146. Tables for true anomaly and perihelion passage in nearly parabolic orbits.

Clissold and Tweddell :
Brown's Nautical Star Chart on the Mercator principle, for use at any time in any part of the world to find the visible stars or to identify an unknown star.
(*Messrs. Brown & Ferguson, Ltd.*) Glasgow, 1935

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Córdoba, Observatorio Nacional Argentino :
Resultados. Vol. 34. Entrega 1, 2. 4to. Córdoba, 1934

Entrega 1. Constantes de placa, formulas y tablas de reduccion para los catálogos astrográficos. Zona $-24°$ a $-31°$.
Entrega 2. Catálogo de 6429 estrellas de repère del catálogo astrográfico reducidas a 1900·0 . . .

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——: Observatory Paper. Nos. 1, 3. 8vo. Typescript, 1935–36
——: Report for the quarter ending 1936 March 31. fol. Typescript, 1936

Coblentz (W. W.) and R. Stair :
*Evaluation of ultraviolet solar radiation of short wave lengths.
(*National Bureau of Standards.*) 8vo. Washington, 1936

Danjon (André) et André Couder :
†Lunettes et Télescopes. Théorie, conditions d'emploi, description, réglage. Préface de A. de la Baume Pluvinel. 8vo. Paris, 1935

Deltheil (R.) :
*Benjamin Baillaud, 1848–1934.
(*Toulouse Observatory.*) 4to. Toulouse, 1935

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Meadowbank, Twickenham : (1) The House : (2) The Observatory ; (3) The Astronomers. (Three articles from the *Richmond and Twickenham Times*.) 1935

†Empire Survey Review :
Vol. III, Nos. 19–21. 8vo. London, 1936

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*Australites, Parts I, II. cr. 8vo. Adelaide, 1934–35

Part I. Classification of the W. H. C. Shaw Collection.
II. Numbers, Forms, Distribution and Origin.

(*G. F. Dodwell.*)

Fox (Philip) :

*Edwin Brant Frost, 1866–1935. 8vo. Chicago, 1936

Gallo (Joaquin) :

Importancia de los estudios solares. (*Universidad Mensual de Cultura Popular*, Febrero 1936.) 4to. Mexico, D.F., 1936
(*The Editor of Universidad Mensual de Cultura Popular.*)

Gallucci (G. P.) :

——: †De Fabrica et Usu Hemisphærii Uranici. fol. Venetiis, 1596
——: †Nova Fabricandi Horaria Mobilia et Permanentia. fol. Venetiis, 1596

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Gifford (A. C.) :

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Hale (George E.) :

——: *The Astrophysical Observatory of the California Institute of Technology.
——: *The Astrophysical Observatory of the California Institute of Technology : The 200-inch Reflector.
(*The Editor of Nature.*) 4to. London, 1936

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